Three Essays on Economics and Incentives

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Dedicated to the paramedics

of the London Ambulance Service

and the staff of the Trauma Unit,

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Also, I would like to give special thanks to

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ABSTRACT

Title of dissertation required

Incentives that encourage us to take particular actions pervade our everyday lives. This thesis investigates incentives in two particular settings: (i) incentives for agents to exert effort when employed, and (ii) the potential for legitimate paid employment to reduce the incentives to commit crime. Chapters 1 and 2 consider setting (i). Chapter 3 considers setting (ii), and is joint work with Prof. Melvyn Coles. All three chapters reach novel conclusions that have policy implications.

Chapter 1 considers the use of Relative Performance Evaluation (RPE) across firms. When the effort of one firm's agent imposes a negative externality on other firms' profits, firms have an incentive to collude to limit agents' efforts. This collusion in incentive contracts leads to new questions; such as, whether compensation consultants provide information which helps to facilitate collusion.

In Chapter 2, equilibrium non-linear incentive contracts are derived, for a situation where firms engage in quantity competition and each firm's output is determined by the unobservable effort of an agent they employ. The potential impact that moral hazard and agency costs have on market outcomes is assessed. Significantly, firms

may fail to make agency cost reducing investments despite the investments being welfare-enhancing.

In the final chapter, agents' decisions regarding job search, crime, consumption and gambling are modelled in a dynamic setting. Notably, the incentive to commit crime is mitigated by an agent's "integrity", or disutility from committing crime. For only a subset of agents does employment status alter the offending decision. My contribution, takes the model as a framework, and analyses self-reported offending data from the Offending, Crime and Justice Survey, 2003-2006. The empirical results highlight the importance of integrity to the offending decision, and that many young adult offenders commit crime whilst employed in low-level jobs.

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Introduction

Incentives are at the heart of economics. They shape the decisions we take throughout our lives. Amongst many things, they shape our decision to search for a job, the level of effort we exert in jobs, and whether or not we commit crime to fund our consumption. The purpose of this thesis is to highlight the impact of incentives within relatively complex environments. The first two chapters focus on the optimal design of incentive contracts for agents, when the firms that employ them are engaged in competitive interactions. The third chapter, co-authored with Prof. Melvyn Coles, highlights how the competing incentives offered by wages, unemployment benefits, the spoils of crime and the criminal justice system are mediated by person-specific characteristics. All three chapters provide novel insights which have policy implications.

Debates over executive pay and incentives are never far from the news, and the literature on contract theory is well established. In Chapters 1 and 2, rather than focusing on the potential strategic commitments offered by incentive contracts, the interplay between a classic moral hazard problem within firms and product market competition is emphasised.

In Chapter 1, the competitive interaction takes a highly reduced form. This simplification enables the otherwise complex problem, of designing contracts involving Relative Performance Evaluation (RPE) across firms, to be solved algebraically.

Generally, it is assumed that Relative Performance Evaluation (RPE) across firms can reduce agency costs in a profit-enhancing way.[1] Chapter 1 highlights an issue which has only received limited attention within the RPE literature: the externalities imposed by one agent's effort on the profits of other firms. This limited attention is despite the presence of such externalities appearing natural, when firms engage in competition.

If an agent's effort imposes a negative externality on rival firms' profits the equilibrium use of RPE can reduce industry profits. Hence, the presence of a negative externality gives firms an incentive to collude when setting incentive contracts; the aim being to limit the effort exerted by agents. For the first time in the economics literature, the possibility of compensation consultants being used as information exchange devices to facilitate collusion is discussed. Additionally, Chapter 1 highlights that, in the presence of effort externalities, the design of incentive contracts may be complex. As such, the encouragement of RPE as "best practice" for firms, by policymakers and investor organisations, could be inappropriate and, in many cases, harmful to shareholders.

Chapter 2 analyses a quantity competition game where an agents' unobservable effort alters the probability distribution of a firm's output (higher effort increases expected output). Simply analysing a quantity competition game where quantities are random variables whilst quantities and prices are restricted to be positive is rare.

[1] RPE is where an agent is rewarded on the basis of their performance relative to the performance of other agents.

The chapter is also novel for being the first work to combine optimal non-linear incentive contracts, where the performance measure is continuous, and product market competition.

Whilst competition means negative effort externalities are still present, the chapter's focus is to consider the impact of moral hazard within firms, and the associated agency costs, on the product market outcome. This reverses the focus of previous papers where the impact of product market interactions on the design of incentive contracts is emphasised. As such, the chapter provides an example demonstrating the importance of opening up the "black box" of firms' profit functions when investigating industrial organisation questions.

For large market sizes, moral hazard reduces expected output more severely than if firms restricted output to jointly maximise profits, i.e. colluded, in the absence of a moral hazard problem. This result demonstrates that the impact of firms' internal workings on market outcomes can be large. However, this result is likely to depend on the number of workers employed by an industry.

More significantly, from a policy perspective, profit-maximising firms may not invest in a perfect monitoring technology even if it is welfare-enhancing. This is because profit-maximising firms fail to consider the positive externality on consumer surplus that reducing agency costs creates. As such, there may be an over-reliance on incentive contracts from a welfare perspective. Also, it suggests that consumers, not simply shareholders, have a legitimate interest in the way companies deal with agency problems.

Chapter 3 analyses incentives from a different perspective. Rather than investigating the optimal design of incentives, it considers how an agent responds to a complex range of incentives when making a range of decisions in a dynamic setting. These decisions concern job search, consumption, crime and gambling. By crime, we mean crimes which have a clear economic motivation, i.e. crimes that can increase an individual's assets or consumption.

This final chapter is split into two parts: sections 3.3 to 3.6 contain a search theoretic model developed by Prof. Melvyn Coles, whilst in sections 3.7 to 3.9 I use the model as a framework for empirical analysis. My empirical analysis uses self-reported offending data from an unusually rich dataset: the Offending, Crime and Justice Survey (OCJS), 2003-2006.

It seems reasonable to assume most citizens prefer well-paid work to committing crime. But what if the chance of finding a job is small and you are broke? Moreover, what if the only work available is in miserable, low-paid jobs? Then, perhaps, crime becomes an attractive option. The optimal choice of crime and job search is found to be essentially a portfolio decision problem which depends on an agent's tastes and opportunities. Significantly, it is shown that agents can be grouped into different types according to their "integrity", or disutility from committing crime, and the wages they obtain in legitimate employment. Agents sort with the aim to specialise in either crime or legitimate employment.

The model highlights that employment status alters the decision to become a criminal for only one group of agents, whom we call the "unfortunates". As one would expect, many other individuals never commit crime regardless of employment status. However, the most interesting agents are those whom we call the "criminally inclined"; these agents, whilst finding job search optimal, commit crime both when unemployed and employed. Also, for criminally inclined agents, gambling yields strictly positive value even though the utility of consumption is assumed to be strictly concave. Gambling has value as it helps agents to specialise in employment or crime.

Additionally, the model incorporates a consumption smoothing problem in the presence of a liquidity constraint. Agents only commit crime once they have run down their stock of assets and the liquidity constraint binds. Combining crime, consumption and gambling represents a first for search models of the labour market.

The OCJS covers England and Wales, and contains uniquely clear proxies for "integrity". These proxies are questions asking respondents for their views regarding the acceptability of committing crime. Chapter 3 represents the first time that OCJS data has been used to study the interplay between the labour market and offending. Various binary choice models, primarily probit specifications, are estimated. Integrity and admitting prior offending each have statistically significant relationships with the probability of offending in the expected directions. Also, the magnitudes of these relationships seem empirically relevant. These findings are consistent with agents sorting into criminal behaviour according to their integrity.

The other, more surprising, finding is that employment status does not have a statistically significant relationship with the probability of offending even after controlling for a wide range of factors. Indeed, the OCJS data, shows that those in low-level occupations have the highest offending rates and that there is a high prevalence of workplace theft. That, amongst the offenders sampled, there were few "unfortunates" and instead many "criminally inclined" agents is plausible given the benign labour market conditions in the period 2003 to 2006.

Overall, this thesis investigates the impact of incentives related to employment. In Chapter 1, new insights are obtained regarding the incentive, created by effort externalities across firms, for firms to collude when setting incentive contracts. In Chapter 2, it is demonstrated that, even when optimal non-linear incentive contracts are used, agency costs can have a significant impact on market outcomes. Also, firms may not invest to reduce agency costs even when it is welfare-enhancing. In contrast, Chapter 3 demonstrates, both theoretically and empirically, that the incentive offered by legitimate paid employment has a smaller impact on deterring crime than is often imagined. This result occurs for two reasons. Firstly, most individuals have a sufficiently high integrity that they never commit crime. Secondly, many of those willing to commit crime do so even whilst employed.

CHAPTER 1

Effort Externalities, Relative Performance Evaluation Across Firms and Collusion in Incentive Contracts

1.1. Introduction

Executive compensation frequently makes the news, particularly, in the aftermath of the Financial Crisis. There are many aspects to this public debate but a common grievance is that executives receive "rewards for luck", i.e. they receive rewards simply for being in charge when an industry experiences a positive shock.[1] Since the early 1980s, it has been argued that Relative Performance Evaluation (RPE) across firms can help reduce such rewards.[2] More importantly, by removing common shocks from agents' performance measures RPE reduces the risk agents bear and, hence, improves the incentive-insurance trade-off which lies at the heart of moral hazard problems. As a result, over the last twenty years, there has been quasi-official encouragement to use RPE. Despite this, only limited theoretical work has been undertaken to analyse the implications of agents evaluated by RPE being located in separate firms.[3] This chapter develops a simple theoretical model to analyse this specific setting. The originality of the work is the inclusion of negative externalities from an agent's effort

[1] For example, see "Bank profits were due to "luck, not skill", Financial Times, 1 July 2009 and "Investment bankers rolling the dice", Financial Times, 5 March 2010.

[2] See Holmstrom (1982).

[3] The original theoretical work, such as Lazear and Rosen (1981), considers a single principal attempting to incentivise multiple agents. The work which has been conducted regarding RPE across firms, such as Fumas (1992), generally does not consider effort externalities.

in one firm on to the profits of other firms. These externalities drive the chapter's main result: firms may have an incentive to collude to restrict agent effort.

In the current model agent effort is unobservable, incentive contracts are linear and the effort of one firm's agent imposes externalities on other firms' profits. Using this model, algebraic solutions for the optimal contract weights are derived and it is shown that placing a negative weight on rival-firm profits is not always optimal. Obtaining algebraic, as opposed to numerical, solutions for the optimal contract weights sets the work apart from Fumas (1992). Algebraic solutions are made possible by the reduced form of firms' profit functions which also stops the incentive contracts having value as strategic commitments.

Incorporating effort externalities between firms' profit functions into an RPE model seems intuitive but appears novel.[4] These externalities are likely to be negative, such as cost-reducing effort in an oligopolistic product market, but may be positive, for example when effort helps develop common technical standards. Significantly, if the externalities are negative, company owners have an incentive to collude to limit agents' efforts. This collusion occurs via incentive contracts. Here, collusion is between principals in separate organisations not between individuals within the same organisation, as is more commonly associated with the principal-agent literature.[5] Once one considers the possibility of collusion via incentive contracts, new questions arise. For example, could compensation consultants be a mechanism for information

[4]Only Fershtman et al (2003) consider RPE-style contracts in the presence of externalities and multiple firms, however, the externalities are located in agents' utility functions rather than in firms' profit functions. The combination of effort externalities between agents and RPE contracts in a single firm with a single principal, has been considered by Itoh (1992) and Choi (1993).

[5]For a discussion of the latter see Tirole (1986).

exchange? Or, could increased reporting requirements for executive compensation help to sustain collusion? One hopes that the general answer is "no", but in specific cases these issues may be relevant.

The presence of externalities also introduces an additional reason, beyond risk-reduction, for linking incentives to the profits of rival firms. The profits of rival firms provide a direct signal of own-agent effort. Indeed, placing a positive weight on rival-firm profits is optimal if there is a sufficiently large positive externality.

Also, the comparative statics for these contract weights depend heavily on context and can be non-monotonic. As such, "one-size-fits-all" recommendations regarding RPE are inappropriate. Highlighting this potential complexity of RPE contracts is valuable considering the encouragement to use RPE. In the UK, Liu and Stark (2009) note this encouragement has come from official reports and bodies representing institutional investors. Whilst in the US, Murphy (2011) notes, increased reporting requirements for executive pay have brought increased transparency to the performance measures used in pay awards.

Alongside this official encouragement, a large empirical literature has developed to investigate whether firms actually use RPE to reward senior executives. Most papers only look for RPE implicitly, i.e. they look for a positive relationship between managerial compensation and own-firm performance and a negative relationship with the performance of other firms. Given the variety of different performance measures, time periods and peer group definitions used, the results are, unsurprisingly, mixed. Papers pointing towards the use of RPE include Antle and Smith (1986), Gibbons and Murphy (1990) and Albuquerque (2009). Papers pointing towards RPE not being used include Barro and Barro (1990), Jensen and Murphy (1990), Janakiraman et al (1992), Aggarwal and Samwick (1999a, 1999b) and Garvey and Milbourn (2003).

More recent studies explicitly identify companies using RPE.[6] Reassuringly, both Gong et al (2011) and Black et al (2011) find that firms claiming to use RPE do have the expected negative relationship between managerial compensation and peer group performance.

Yet, these studies report a minority of US firms explicitly stating the use of RPE. Within the S&P 500 estimates of RPE use vary from 17.3% to 37.8% of firms.[7] RPE use also varies by industry. De Angelis and Grinstein (2010) found that 68% of energy and utility firms used RPE, whereas only 15% of retail firms and 17% of business equipment firms used RPE. These results are consistent with the model's findings that only some firms, in some industries, will find RPE with a negative weight on rival-firm performance optimal.

The paper continues with a literature review in section 1.2 before the model is introduced in section 1.3. Section 1.4 solves the model when there are negative effort externalities and section 1.5 considers the possibility of collusion via incentive contracts. In section 1.6 positive effort externalities are considered before section 1.7 concludes.

[6] Such studies include Carter et al (2009), De Angelis and Grinstein (2010), Gong et al (2011) and Black et al (2011).

[7] The figure for Black et al (2011) is 17.3%, for De Angelis and Grinstein (2010) the figure is 34% and for Gong et al (2011) it is 37.8%. In the UK, perhaps reflecting the longer period of encouragement to use RPE, Carter et al (2009) found 51.2% of FTSE350 firms used some form of RPE when making equity grants in 2005.

When describing the model, the term RPE refers to any incentive contract which places a weight, positive or negative, on the profit of other firms. Rewarding agents solely by assessing own-firm profits is referred to as Absolute Performance Evaluation (APE).

1.2. Theoretical Literature

1.2.1. Overview

The literature on RPE has its origins in the literature on tournaments.[8] However, this literature focuses on the problem of a principal incentivising multiple agents within a single firm. On the other hand, the strategic delegation literature considers the value of incentive contracts as strategic commitments within product market competition. However, this literature, beginning with Vickers (1985), Fershtman and Judd (1987) and Sklivas (1987), does not incorporate risk-averse agents. As such, the original motivation for using RPE, to reduce agency costs is missing.

The current chapter focuses on the original risk-reduction motivation for RPE in a multi-firm setting. The earlier papers which do combine RPE, agency costs and competition between firms, such as Fumas (1992), emphasise the trade-off between writing contracts to incentivise effort and writing them for strategic reasons. They do not include effort in the competitive interaction itself. Hence, in contrast to the current chapter, these earlier papers do not consider effort externalities. Similarly, whilst cartel theory does consider the role of incentive contracts in supporting collusion, it has not considered collusion in effort levels.

[8]See Lazear and Rosen (1981).

Finally, this chapter contributes to the literature explaining the empirical reality of only some firms using RPE. Here, the chapter is similar to Fumas (1992) and Aggarwal and Samwick (1999b) in emphasising the importance of the product market in contracting decisions. The other main explanations relate to the managerial labour market and the subversion of the pay-setting process by managers themselves.

1.2.2. Principal-agent theory

Lazear and Rosen (1981) introduce the central idea in favour of RPE/tournaments: if there is a common component to shocks, by comparing the performance of agents, the common element of risk can be removed from pay schemes whilst effort incentives are preserved. Other early papers in this literature include Holmstrom (1982), Green and Stokey (1983) and Nalebuff and Stiglitz (1983b). More recent papers involving RPE within a single firm consider more complex contracting scenarios. For example, Athey and Roberts (2001) consider how combining a project selection choice with a hidden action problem can affect the design of organisations.

Indeed, a small number of papers considering RPE where effort externalities exist between agents in a firm's production function. Crucially, these papers only consider cases where there is a single principal. Hence, the possibility of collusion between principals is not investigated. In these papers, the notion of "sabotage" between agents, as explored by Lazear (1989), bears some similarity to the negative effort externalities explored in this chapter. Both Lazear (1989) and the current chapter highlight that if the incentives offered to agents can be co-ordinated, profits may be increased by blunting the strength of incentives to exert effort. A difference between this chapter and Lazear's work is that a single principal would always prefer a lower level of "sabotage". In the current framework of effort externalities between firms, a principal acting independently is unconcerned about the impact of their agent's effort on rival firms' profits. The interesting question in the current chapter is whether, and how, separate principals can co-ordinate their incentive contracting decisions.

Itoh (1992) and Choi (1993) both consider RPE combined with positive effort externalities[9] when two agents performs two tasks within a single firm. The present work is complementary. The current chapter considers the case where the incentives offered to the agents cannot be co-ordinated. By considering a single principal, Itoh (1992) and Choi (1993) investigate the case where incentives can be perfectly co-ordinated. The present chapter's results regarding the complexity of setting the optimal contract weights are similar to those in these earlier papers. However, the present chapter further emphasises this complexity by showing that the partial derivatives of the contract weights can be non-monotonic. This leads on to an evaluation of the quasi-official encouragement of RPE across firms which the earlier theoretical literature does not consider.

Regarding RPE across firms, Holmstrom (1982) notes that the observation of firms beginning to reward executives using RPE, could be supported by his theory. However, he does not develop this point. Also, Hart (1983) and Nalebuff and Stiglitz (1983a) suggest that the outcome of product market competition reflects the relative performance of managers. Hence, APE contracts based on own-firm profits reflect

[9] Itoh (1992) does allow for negative effort externalities, although, this is not the emphasis.

relative performance in a crude fashion. Whilst this is true, it does not address the main theoretical reason for using contracts explicitly containing RPE across firms: to insure agents against industry-wide shocks.[10]

1.2.3. Strategic delegation

Beginning with Vickers (1985) a number of papers consider the strategic value of RPE contracts. These papers include: Miller and Pazgal (2001), Jansen et al (2009) and Chirco et al (2011). In quantity competition, compared to APE, RPE causes the agent's reaction function to pivot upwards leading to more aggressive output choices. However, as discussed above, an incentive-insurance trade-off is absent from these papers.

As a variant on the RPE-strategic delegation theme, Kockesen et al (2000) and Miller and Pazgal (2002) view the strategic commitment as concerning whether the agents employed have a preference for high pay in an absolute sense, or, in a relative sense. Fershtman et al (2003) take a similar idea and include a moral hazard problem regarding unobservable effort. However, in contrast to the current chapter, the direct effort externality in Fershtman et al's (2003) paper affects the utility received by agents. This difference appears important, as Fershtman et al (2003) find it is always optimal to place a non-negative weight on rival-firm output in the incentive contract offered. Unlike the present model, there is value in insuring agents against the idiosyncratic shocks affecting other firms.

[10] A range of papers considers the interplay between moral hazard, product competition and incentive contracts using APE. Discussion of this more general literature occurs in Chapter 2.

1.2.4. Competition, moral hazard and RPE

The main paper combining strategic delegation, agency costs and RPE is Fumas (1992). However, in Fumas's work, agent i's effort only affects the fixed costs of firm i.[11] As such, effort externalities are not present and so there is no direct reason for firms to collude to limit the effort exerted.

Fumas finds that a trade-off between the strategic and risk-reduction properties of RPE only exists in the case of strategic complements, i.e. price competition. Here, if agency costs are sufficiently low, a positive weight should be placed on rival-firm profits. This parallels the qualitative differences between the results for negative and positive externalities in the present model. However, in the current paper, any trade-off is between using rival-firm profits to reduce the variance of an agent's pay and using them as a direct signal of agent effort. Lastly, as with Aggarwal and Samwick (1999b), Fumas notes that RPE may reduce industry profits despite it being individually rational for firms to use it. However, neither paper discusses methods to overcome this prisoners' dilemma.

Lamirande et al (2008) do extend Fumas's (1992) model to include collusion regarding the choice between APE and RPE. In Lamirande et al (2008), the desirability of colluding is conditional on the magnitude of the agency cost that RPE can avoid being sufficiently small. In the present chapter, collusion to not use RPE is conditional on the negative effort externality being sufficiently large. More importantly,

[11] The same structure is used by Aggarwal and Samwick (1999b).

when a negative effort externality exists, the incentive for firms to collude in the setting of contract weights is unconditional.[12]

Regarding effort externalities themselves, both Gibbons and Murphy (1990) and Janakiraman et al (1992) mention them as possibilities. However, in Gibbons and Murphy the comment remains in the spirit of the team production literature.[13] The comment refers to positive production externalities between agents in the same firm. Janakiraman et al (1992) note the potential trade-off between incentivisation and risk-reduction when setting the weight on rival-firm profits in an RPE contract. However, the point is made to differentiate between two empirical tests of the RPE hypothesis. Janakiraman et al (1992) do not develop a full model to investigate this issue.

1.2.5. Incentive contracts and collusion

Papers considering the potential influence of incentive contracts on managers' incentives to collude include Spagnolo (2000, 2005), Chen (2008) and Aubert (2009). These papers make the intuitive argument that linking pay to profits encourages managers to collude. In particular, Spagnolo (2000) shows that if stock-related compensation can induce managers to collude, then in an infinitely repeated setting it is a subgame perfect Nash equilibrium for company owners to offer these contracts. However, these papers do not consider the possibility that principals could want to collude over the actual effort levels induced in agents. This novel idea provides an

[12]Lamirande et al (2011) consider collusion over contract weights, but only in the absence of agency costs. Additionally, Asseburg and Hofmann (2010) and Guigou et al (2007) extend Fumas (1992) in other directions. Asseburg and Hofmann relax the assumption of perfectly correlated random shocks, whilst Guigou et al allow entry of additional firms.

[13]For example, see Holmstrom (1982).

alternative explanation for industries lacking competition being associated with low productivity.[14] Rather than weak incentives to exert effort being the consequence of low levels of competition, the present model suggests that restricting effort could be the very mechanism by which product market collusion is achieved.

1.2.6. Explanations for RPE's mixed empirical results

A number of theoretical papers aim to explain the mixed empirical results for the use of RPE. As already noted the present paper and Aggarwal and Samwick (1999b) suggest product market competition as an explanation. Most other papers suggest that pressures from the managerial labour market can explain the observed lack of RPE. For example, Himmelberg and Hubbard (2000) suggest that the marginal value of managerial talent/effort, and hence the amount a firm is willing to pay managers, increases with firm size. In turn, firm size is positively correlated with common industry shocks. So managerial compensation shows a positive correlation with industry shocks. A similar concept, although not directly linked to firm size, is provided by Celentani and Loveira (2006).

Oyer (2004) suggests that pay awards positively correlated with industry performance help to retain managers. Alternatively, Garvey and Milbourn (2003) argue that managers' private asset holdings provide insurance against common profit shocks, and therefore explicit RPE is not required.

[14] For example, see Nickell (1996).

Lastly, a particularly prominent explanation, put forward by Bebchuk and Fried (2003), is the "managerial power" hypothesis. This suggests the pay setting process is, itself, subject to agency problems and has been captured by managers seeking to maximise their own rewards.

1.3. The Model

Consider two firms, or principal-agent pairs, each consisting of a risk-neutral owner (principal) and a risk-averse manager (agent). Assume both principals and agents have full information regarding both pairs' characteristics.

The model consists of three stages:

Stage 1 - The principals choose between APE and RPE.

Stage 2 - Given the contract structure chosen in stage 1 each principal selects the optimal contract weights to use.

Stage 3 - Based on the incentive contracts offered each agent selects their optimal effort level.

In each stage the participants take their decisions independently and simultaneously. At the end of stage 3 the values of the shock terms are realised and all players receive their pay-offs. Whilst modelling the choice between APE and RPE as a separate stage game is somewhat artificial, it is chosen for two reasons. Firstly, it helps to illustrate whether firms have an incentive to collude to not use RPE. Secondly, it reflects the fact that the decision between APE and RPE is potentially a strategic commitment. This commitment power results from the legally binding nature of the compensation scheme details stated in proxy statements/annual reports.

Each principal maximises their expected profit from the interaction with the rival firm, less the expected cost of the payment made to their own agent:

$$E(PP_i) = E(\pi_i(e_i, e_j, \varepsilon_i)) - E(T_i)$$

Here π_i is firm i's gross profit, e_i and e_j are the efforts exerted by agents i and j respectively, ε_i is firm i's idiosyncratic shock term and T_i is the transfer payment from principal i to agent i.

Two profit functions are considered: (i) where an agent's effort imposes a negative externality on the rival firm's profit and (ii) where an agent's effort generates a positive externality. In the case of a negative externality, let firm i's profit function be:

$$\pi_i(e_i, e_j, \varepsilon_i) = F + be_i - ce_j + \varepsilon_i \tag{1.1}$$

F, b and c are all exogenous and strictly positive. Also, it is assumed that $b > c$. As such, agent i's effort has a greater marginal impact on firm i's profits than the externality imposed by a unit of agent j's effort. This seems a natural assumption to make.

For the case of a positive effort externality let firm i's profit function be:

$$\pi_i(e_i, e_j, \varepsilon_i) = F + be_i + de_j + \varepsilon_i \tag{1.2}$$

Here d is an exogenous parameter satisfying $b > d > 0$. Restricting (1.1) and (1.2) to not contain interaction terms between e_i and e_j allows algebraic solutions for the optimal contract weights to be obtained.[15]

Whilst ε_i and ε_j are firm-specific, they can be correlated. Let ε_i be normally distributed such that $\varepsilon_i \sim N(0, \sigma_i^2)$ and, similarly, let ε_j be defined as $\varepsilon_j \sim N(0, \sigma_j^2)$. Let the covariance between the shock terms be $cov(\varepsilon_i, \varepsilon_j) = \rho\sigma_i\sigma_j$, where $\rho \in [0, 1]$ is the correlation coefficient.[16] For simplicity assume the magnitudes of the variances for ε_i and ε_j are identical, i.e. $\sigma_i^2 = \sigma_j^2 = \sigma^2$. This means the covariance reduces to $cov(\varepsilon_i, \varepsilon_j) = \rho\sigma^2$. Restricting ρ to be positive implies that firms' profits respond in the same direction to particular shocks. When $\rho = 0$, the firms' shock terms are uncorrelated and RPE cannot reduce the variance of the agent's transfer payment. When $\rho = 1$, the firms face a common shock.

Agent i selects effort, e_i, from the range $e_i \in [0, \overline{e}]$ where $\overline{e}$ is sufficiently large to not affect the equilibrium outcomes. Agent i maximises their expected utility:

$$EU_i = E[u(T_i - g(e_i))] \tag{1.3}$$

[15]If a profit function including an interaction term is used, e.g.:

$$\pi\left(e_i, e_j, \varepsilon_i\right) = (F + be_i - ce_j)e_i + \varepsilon_i,$$

then, when RPE is used, each principal's problem in stage 2, involves solving a pair of sixth-order polynomials. As such, algebraic solutions to the optimal contract weights cannot be described. The same issue is encountered by papers using Fumas's (1992) model. Appendix 1.2 explains this issue in detail. The current chapter follows the approach of Itoh (1992) and Choi (1993) in restricting the output variable to being a linear combination of the agents' efforts.

[16]A negative correlation between the shock terms, $\rho \in [-1, 0)$, does not alter the main qualitative findings regarding when a prisoners' dilemma involving the use of RPE may occur. However, a negative correlation coefficient does alter the optimal contract weights and effort levels.

where $u(.)$ is the agent's utility function and $g(.)$ is the cost of effort function. For simplicity assume $g(e_i) = e_i^2$.[17] For tractability, the LEN-framework of Holmstrom and Milgrom (1987) is followed. The LEN-framework involves a linear incentive contract, exponential utility function and a performance measure with a normally distributed shock term. Following Holmstrom and Milgrom, the agent's utility function has the form $u_a(y) = -e^{-Ry}$, where y is the agent's pay-off and R is the coefficient of absolute risk aversion. Each agent is assumed to be risk-averse and so $R > 0$. The form of (1.3) along with the exponential utility function means that, to maximise expected utility, an agent simply has to maximise their certainty equivalent. A proof of this equivalence result is provided in Appendix 1.1.

As analysis is restricted to linear contracts the APE incentive contract takes the form:

$$T_i = w_i + \alpha_i \pi_i \tag{1.4}$$

where w_i is a flat wage and α_i is a weight to be determined. The RPE incentive contract takes the form:

$$T_i = w_i + \alpha_i \pi_i + \beta_i \pi_j \tag{1.5}$$

where α_i and β_i are weights to be determined. Note the APE contract is simply the RPE contract with the constraint $\beta_i = 0$ imposed. If the strategic commitment possibilities of the APE/RPE decision are ignored, the nesting of (1.4) within (1.5) means that, whenever principal i's optimal solution involves $\beta_i \neq 0$, RPE must offer a higher pay-off to the principal than APE.

[17] The structure of (1.3) implies the cost of effort is measured in monetary units.

In stage 2 principal i chooses w_i, α_i and, if relevant, β_i to maximise $E(PP_i)$. In stage 1 the principals choose from $D \in \{APE, RPE\}$ by comparing the values of $E(PP_i)$. The overall game is solved from the last stage backwards with the aim being to find the equilibrium contract choice in stage 1. As will become clear, the reduced form profit functions (1.1) and (1.2) imply that the contract weights selected by firm i in stage 2 are invariant to the contract weights chosen by firm j. As such, for these profit functions, the notion of a Nash equilibrium in the stage 2 subgame is trivial.[18]

1.4. Solving the Model - Negative Effort Externalities

Here profits are given by (1.1). To analyse the principals' stage 1 decisions, stages 2 and 3 need to be considered following each of the possible action pairs: (APE, APE), (RPE, RPE), (RPE, APE) and (APE, RPE). Following each action pair, the solution method is fundamentally the same. As a result, a full description of the solution method is only provided for the case of (RPE, RPE).

1.4.1. Solution following (RPE, RPE)

Stage 3

Agent i selects e_i to maximise their certainty equivalent. Using the RPE incentive contract and (1.1), agent i's transfer payment is:

$$T_i = w_i + \alpha_i (F + be_i - ce_j + \varepsilon_i) + \beta_i (F + be_j - ce_i + \varepsilon_j)$$

The variance of this transfer payment is:

[18] As discussed in Appendix 1.2, finding algebraic solutions is difficult when the profit functions lead to firm i's contract weights depending on those of firm j.

$$(1.6) \qquad Var(T_i) = \alpha_i^2\sigma^2 + \beta_i^2\sigma^2 + 2\alpha_i\beta_i\rho\sigma^2$$

So agent i's maximisation problem is:

$$\max_{e_i} CE_i = w_i + \alpha_i (F + be_i - ce_j) + \beta_i (F + be_j - ce_i)$$

$$-e_i^2 - \frac{R}{2}\sigma^2 \left(\alpha_i^2 + 2\rho\alpha_i\beta_i + \beta_i^2\right)$$

By inspection, this problem is concave in e_i. Hence, re-arranging the problem's first-order condition (FOC) gives agent i's optimal effort as:

$$(1.7) \qquad e_i = \frac{1}{2}(\alpha_i b - \beta_i c)$$

By symmetry, $e_j = \frac{1}{2}\left(\alpha_j b - \beta_j c\right)$. Note each agent's optimal effort is independent of the other agent's effort and depends solely on the contract weights selected by their own principal. This results from the linear profit functions being used and means it is trivial that an equilibrium exists in the stage 3 subgame. Using the expressions for e_i and e_j, firm i's expected profit, gross of agent i's expected transfer payment, is:

$$(1.8) \qquad E(\pi_i) = F + \frac{1}{2}\left(b^2\alpha_i + c^2\beta_j - bc\alpha_j - bc\beta_i\right)$$

Stage 2

Principal i's problem is:

$$\max_{w_i,\alpha_i,\beta_i} E(PP_i) = E(\pi_i) - E(T_i)$$

subject to the incentive compatibility constraint (ICC):

$$\arg\max_{e_i} CE_i = w_i + \alpha_i \left(F + be_i - ce_j\right) + \beta_i \left(F + be_j - ce_i\right)$$

$$-e_i^2 - \frac{R}{2}\sigma^2 \left(\alpha_i^2 + 2\rho\alpha_i\beta_i + \beta_i^2\right)$$

and the participation constraint (PC):

$$CE_i = w_i + \alpha_i \left(F + be_i - ce_j\right) + \beta_i \left(F + be_j - ce_i\right) \tag{1.9}$$

$$-e_i^2 - \frac{R}{2}\sigma^2 \left(\alpha_i^2 + 2\rho\alpha_i\beta_i + \beta_i^2\right) \geq \widehat{u}$$

where $\widehat{u}$ is the agent's reservation utility. For simplicity, assume the labour market is perfectly competitive and $\widehat{u}$ is the utility obtained from outside offers.

It is straightforward to argue that (1.9) must bind with equality. Setting $E(T_i) > \widehat{u}$ cannot be rational. If the principal sets $E(T_i) = \widehat{u}$, the agent will still participate but $E(T_i)$ is strictly lower. Hence, setting $E(T_i) = \widehat{u}$ strictly increases the principal's pay-off compared to setting $E(T_i) > \widehat{u}$. Since (1.9) binds with equality:

$$\begin{aligned} E(T_i) &= w_i + \alpha_i \left(F + be_i - ce_j\right) + \beta_i \left(F + be_j - ce_i\right) \\ &= \widehat{u} + e_i^2 + \frac{R}{2}\sigma^2 \left(\alpha_i^2 + 2\rho\alpha_i\beta_i + \beta_i^2\right) \end{aligned}$$

Using this value for $E(T_i)$ and (1.8), the principal's problem can be written as the following unconstrained optimisation problem:

$$\max_{\alpha_i,\beta_i} E(PP_i) = F + \frac{1}{2}\left(b^2\alpha_i + c^2\beta_j - bc\alpha_j - bc\beta_i\right) - \widehat{u} \tag{1.10}$$

$$-\left(\frac{1}{2}\left(\alpha_i b - \beta_i c\right)\right)^2 - \frac{R}{2}\sigma^2\left(\alpha_i^2 + 2\rho\alpha_i\beta_i + \beta_i^2\right)$$

This problem is shown to be concave in Appendix 1.1.

In (1.10) α_i and β_i are additively separable from α_j and β_j. Hence, the optimal values of α_i and β_i are invariant with respect to α_j and β_j. As such, it is trivial that an equilibrium exists in the stage 2 subgame. The two FOCs for the problem in (1.10) are:

$$\frac{\partial E\left(PP_i\right)}{\partial \alpha_i} = \frac{1}{2}b^2 - \frac{1}{2}b^2\alpha_i + \frac{1}{2}bc\beta_i - R\sigma^2\alpha_i - R\sigma^2\rho\beta_i = 0$$

$$\frac{\partial E\left(PP_i\right)}{\partial \beta_i} = \frac{1}{2}bc\alpha_i - \frac{1}{2}bc - \frac{1}{2}c^2\beta_i - R\sigma^2\beta_i - R\sigma^2\rho\alpha_i = 0$$

Solving these FOCs as a pair of simultaneous equations gives the optimal contract weights. The optimal weight for own-firm profits is:

$$\alpha^R = \frac{b\left(b + c\rho\right)}{2R\sigma^2(1-\rho^2) + b^2 + c^2 + 2bc\rho}$$

and the optimal weight for rival-firm profits is:

$$\beta^R = -\frac{b\left(c + b\rho\right)}{2R\sigma^2(1-\rho^2) + b^2 + c^2 + 2bc\rho}$$

Since $\rho \in [0,1]$, it is always the case that $\alpha^R > 0$ and $\beta^R < 0$.[19] Here RPE is used in the "classic" sense. A negative weight is placed on rival-firm profits to reduce the risk borne by agents.[20] As such, using RPE reduces the agency costs borne by the principal.

Despite the signs of α^R and β^R being fixed, Appendix 1.6 shows the signs of the comparative statics of α^R and β^R can vary depending on the precise values of b, c, and ρ. Indeed, Appendix 1.6 shows that when,

$$c^2 + 2R\sigma^2 < b^2 < \left(\frac{c^2 + 4R\sigma^2}{c}\right)^2,$$

the relationship between α^R and ρ may be non-monotonic. Also, the relationship between the contract weights and a number of other exogenous parameters can be non-monotonic in specific circumstances. This finding of non-monotonic comparative statics parallels the work of Asseburg and Hofmann (2010). These results suggest that setting contract weights when using RPE may not be straightforward.

This complexity appears to result from the trade-off between using RPE to reduce transfer payment variance and using rival-firm profits as a direct signal of agent effort. When $\rho = 0$, RPE has no risk reduction properties[21], and the only derivatives whose signs vary are those of β with respect to b and β^{Col} with respect to c. When

[19]When $\rho \in [-1,0)$, β^R may be positive since $b > c$.

[20]When $\rho = 1$, i.e. firms experience a common shock, the optimal incentive weights reduce to $\alpha^{RR} = \frac{b}{b+c}$ and $\beta^{RR} = -\frac{b}{b+c}$. Here RPE is perfect, in that any variance is removed from the agent's transfer payment. In this special case, there are no agency costs associated with providing incentives.

[21]Here, rewarding agent i on the basis of π_j actually increases the variance of agent i's transfer payment.

$\rho = 1$, RPE fully removes transfer payment risk, and all the derivatives have signs independent of the exogenous parameters. Also, note that the optimal values of α and β are inherently linked if RPE is playing a risk reduction role.[22]

Inserting α^R and β^R back into (1.7) gives agent i's optimal effort as:

$$e^R = \frac{b\left(b^2 + 2\rho bc + c^2\right)}{2\left(2R\sigma^2(1-\rho^2) + b^2 + c^2 + 2bc\rho\right)}$$

By symmetry, α^R, β^R and e^R are also the optimal values for principal-agent pair j. Inserting e^R, α^R and β^R back into (1.9) and remembering that $CE_i = \widehat{u}$ gives the optimal value of w_i (denoted w^{RR}). The expression for w^{RR} and the other optimal wage expressions are shown in Appendix 1.1. The comparative statics for e^R are straightforward and are again shown in Appendix 1.6. As b increases, so that the marginal return to effort increases, and c increases, so that π_j becomes a more informative signal of agent i's effort, the optimal effort level, e^R, increases. When the correlation between the profit shocks, ρ, increases or the agent's absolute risk aversion, R, decreases or the variance of profits, σ^2, decreases, the optimal effort level again increases. For these changes in ρ, R and σ^2 the increase in effort results from reduced agency costs.

Since the optimal values of α_i, β_i and e_i are invariant with respect to α_j, β_j and e_j, and vice versa, there can be no strategic benefit from committing to use APE or RPE.[23] Committing to set $\beta_i = 0$ has no impact on the contracting choices of

[22] Considering (1.6), the variance of the agent's transfer payment is minimised when $\beta_i = -\alpha_i$.

[23] This outcome results directly from the absence of interaction terms involving the agents' efforts. (Compare this section's results with those in Appendix 1.2)

principal j. This, combined with β^R being non-zero, means it is always individually rational for a principal to use RPE. Hence, playing RPE in the stage 1 subgame is a strictly dominant strategy and the Nash equilibrium is (RPE, RPE). The case when both firms select APE is nevertheless analysed to demonstrate that a prisoners' dilemma can exist regarding the use of RPE.

Inserting α^R and β^R into the objective function of (1.10) gives the expected pay-off following (RPE, RPE) as:

$$E(PP^{RR}) = F + \frac{b\,(b-2c)\,(b^2+2\rho bc+c^2)}{4\,(2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho)} - \widehat{u}$$

1.4.2. Solutions following other stage 1 action pairs

Following (APE, APE)

When APE is used, the problems facing the agent and the principal are the same as above but with the restrictions $\beta_i = 0$ and $\beta_j = 0$ imposed. A brief description of the solution method is given in Appendix 1.1. The optimal value of α_i under APE is:

$$\alpha^A = \frac{b^2}{b^2+2R\sigma^2}$$

and the optimal effort level is:

$$e^A = \frac{b^3}{2\,(b^2+2R\sigma^2)}$$

As expected, as b is increased α^A and, in turn, e^A increase. When the agent's absolute risk aversion, or the variance of profits, increase, α^A and e^A decrease. Note, in contrast to the comparative statics for RPE, the comparative statics for APE are simple and intuitive. For APE, even when $\rho \in (0,1)$, the signs of the partial derivatives never change.

It is straightforward to demonstrate that $e^R > e^A$ so that effort is always higher under RPE. Whilst in most principal-agent settings this demonstrates the desirability of RPE, here, the negative effort externality means things are more complicated. As RPE results in increased effort, the externality imposed on the rival firm also increases. This drives the result that principals may face a prisoners' dilemma when choosing RPE over APE.

Each principal's expected pay-off following (APE, APE) is:

$$E\left(PP^{AA}\right) = F + \frac{b^3\left(b-2c\right)}{4\left(2R\sigma^2+b^2\right)} - \widehat{u}$$

Following (RPE, APE) **and** (APE, RPE)

Appendix 1.1 includes a description of how the expected pay-offs following (RPE, APE) and (APE, RPE) are derived. A pay-off matrix representing the stage 1 subgame, when the APE/RPE decision has commitment power, is shown in Figure 1.1 in Appendix 1.1.

1.4.3. A Prisoners' Dilemma

As discussed in section 1.4.1, it is always individually rational for a principal to select RPE in stage 1. This section highlights that a prisoners' dilemma can occur when the APE/RPE decision has commitment power.

Proposition 1.1 *A prisoner's dilemma occurs in the stage 1 subgame, with* (APE, APE) *being Pareto superior to* (RPE, RPE)*, if:*

$$c < b < 2c$$

Proof. (RPE, RPE) offers higher pay-offs for both principals than (APE, APE), and is therefore Pareto superior, only if $E\left(PP^{RR}\right) > E\left(PP^{AA}\right)$.[24] This inequality reduces to:

$$E\left(PP^{RR}\right) - E\left(PP^{AA}\right) = \frac{Rb\sigma^2\left(b-2c\right)\left(c+b\rho\right)^2}{2\left(2R\sigma^2+b^2\right)\left(2R\sigma^2\left(1-\rho^2\right)+b^2+c^2+2bc\rho\right)} > 0$$

This inequality only holds if $b > 2c$.[25] Combining this with the starting condition $b > c$, when $c < b < 2c$ a prisoners dilemma exists. When $c < b < 2c$ the Nash equilibrium is to play (RPE, RPE) but both principals would receive higher expected pay-offs following the action pair (APE, APE). ■

That a prisoners' dilemma only occurs when $b < 2c$ is intuitive. Only when the marginal return to own-agent effort, b, is sufficiently large compared to the effort

[24]Since one principal always prefers (RPE, RPE) to (RPE, APE) or to (APE, RPE), these latter two action pairs cannot represent Pareto improvements over (RPE, RPE).

[25]Note this condition is independent of the correlation coefficient ρ. This indicates that: (i) the effort externality in the profit functions, rather than the correlation of errors, drives the prisoners' dilemma, and (ii) the prisoners' dilemma will not disappear if the correlation coefficient is negative.

externality, i.e. $b > 2c$, does the increased effort resulting from RPE lead to higher industry profits. When $b < 2c$, the gain in profits associated with increased own agent effort is more than offset by the reduction in profits resulting from the extra effort exerted by the rival firm's agent.

As such, when $c < b < 2c$, both principals would be better off if they could agree to each play APE instead of RPE. Thus the principals have a motivation to collude in the setting of incentive contracts. It seems reasonable to assume that a formal contractual agreement to use only APE would fall foul of antitrust law.[26] This means it is worth investigating whether firms can collude to use only APE when formal contracts and side-payments are unavailable. Of course, such collusion is not sustainable in a single period setting. Since the Nash equilibrium in stage 1 is (RPE, RPE), a commitment to play (APE, APE) is not credible. Holding the action of the rival firm fixed, a firm can increase their pay-off by switching from APE to RPE, i.e. a profitable deviation from playing APE exists. However, Section 1.5 shows collusion to use only APE can be sustainable in an infinite period game. In an infinite period setting punishments for defections are possible.

Yet, when $c < b < 2c$, the components of the expected pay-offs, $E\left(PP^{AA}\right)$ and $E\left(PP^{RR}\right)$, attributable to effort are negative. As a result, in this model, when principals have an incentive to collude to use only APE, they could achieve higher profits by colluding to set $e_i = e_j = 0$. Section 1.5 shows firms achieve the highest pay-offs by colluding in the actual values of α and β set. In this latter case, where the

[26] However, in the US an exemption from antitrust law can be obtained if the contractual conditions resulted from a collective bargaining process with a labour organisation. See Edelman and Doyle (2009) for a discussion of this issue in relation to sports teams.

effort externality is fully internalised, it remains optimal for firms to induce strictly positive levels of effort.

The condition for (RPE, RPE) to be Pareto optimal, $b > 2c$, is notable for its independence from agents' risk aversion and the variance of profits. This feature is specific to the symmetric setting considered. Appendix 1.3 considers a case of asymmetric profit functions.

1.5. Collusion in Incentive Contracts

1.5.1. Contract weight collusion

Section 1.4 demonstrates that principals may benefit from an agreement to use only APE. This section demonstrates that firms achieve the highest profits when colluding to set the contract weights α and β. The critical discount factors, δ, for contract weight collusion and APE collusion to be sustainable are derived. The following description is for contract weight collusion.

Firstly, the expected pay-offs when principals collude in the setting of α and β need to be derived. For simplicity, assume δ is common to both principals.

Lemma 1.1 *Under contract weight collusion, the optimal weight to place on own-firm profits is:*

$$\alpha^{Col} = \frac{(b-c)(b+c\rho)}{2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho}$$

and the optimal weight to place on rival-firm profits is:

$$\beta^{Col} = -\frac{(b-c)(c+b\rho)}{2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho}$$

The optimal effort for each agent to exert is:

$$e^{Col} = \frac{(b-c)(b^2+c^2+2bc\rho)}{2(2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho)}$$

Proof. See Appendix 1.4.

Note, when collusion occurs in contract weights, β^{Col} is negative. As such, under contract weight collusion, RPE is still used to reduce agency costs. This result means that, even when RPE involving a negative weight on rival-firm profits is observed, collusion in incentive contracts cannot be ruled out.[27]

Given the negative effort externalities are internalised by collusion, it is unsurprising that the magnitudes of α^{Col} and β^{Col} are lower than α^R and β^R. These weaker incentives lead to a lower optimal effort level, i.e. $e^{Col} < e^R$.

Lemma 1.2 *The principals' pay-offs from contract weight collusion are always Pareto superior to the pay-offs following* (APE, APE) or (RPE, RPE)*.*

Proof. See Appendix 1.4.

[27]Interestingly, when $\rho = -1$, the expressions for the optmal contract weights and effort level reduce to: $\alpha^{Col} = 1$, $\beta^{Col} = -1$ and $e^{Col} = \frac{b-c}{2}$.

Despite $E\left(PP^{ColCol}\right)$ always offering the highest pay-off, neither principal would unilaterally select the collusive contract weights, α^{Col} and β^{Col}. Noting there is no strategic benefit from setting α^{Col} and β^{Col}, the optimal contract weights to select unilaterally must remain α^{R} and β^{R}. When each principal acts independently they could select α^{Col} and β^{Col}; however, the solution to their optimisation problem in Section 1.4.1 is α^{R} and β^{R}. By revealed preference using α^{R} and β^{R} must give a higher expected pay-off than α^{Col} and β^{Col}.

Now the sustainability of contract weight collusion needs to be established. Assume the principals' contract weight setting game is repeated for an infinite number of periods.[28] Friedman (1971) shows that for a sufficiently high δ, a "Grim-Trigger" strategy allows collusion to be sustained in an infinitely repeated game (supergame).[29] Adapting Friedman's notation, let the strategy played by principal i in the infinitely repeated game be Z_i and define the strategy vector of the two principals as $Z = (Z_i, Z_j)$. Denote the strategy played by principal i in period t as s_{it} and define Z_i as follows:

$s_{i1} = Collude,$

$s_{it} = Collude$ if $s_{i\tau}, s_{j\tau} = Collude$ where $\tau = 1, ..., t-1$ and $t = 2, 3, ...$

$s_{it} = Defect$[30] otherwise.

In words, principal i plays $Collude$ in period 1 and will continue to play $Collude$ in each period t as long as neither principal plays $Defect$ in any period prior to period

[28] As Tirole (1988) describes, this assumption can be replaced by an assumption of a finite number of periods but with uncertainty regarding when the game will end.

[29] However, collusion is not the only equilibrium strategy in the supergame. Playing the single-period Nash equilibrium strategy in every period is also a subgame perfect Nash equilibrium.

[30] To "$Defect$" means the principal selects the contract weights which are optimal when the principal acts unilaterally, i.e. α^{R} and β^{R}.

t. If either principal plays $Defect$, then in each subsequent period principal i plays $Defect$. Define Z_j in a similar fashion. Friedman (1971) provides a proof that Z will be a subgame perfect Nash equilibrium if:

$$\sum_{t=0}^{\infty} \delta^t E\left(PP^{ColCol}\right) > E\left(PP^{RCol}\right) + \sum_{t=1}^{\infty} \delta^t E\left(PP^{RR}\right)$$

This inequality states that collusion is a subgame perfect Nash equilibrium, if the discounted stream of expected pay-offs from contract weight collusion exceeds the expected pay-off from unilaterally defecting to use α^R and β^R in the current period, and, in all subsequent periods, both firms setting α^R and β^R. By symmetry, this inequality is identical for both principals.

Proposition 1.2 *Contract weight collusion is sustainable in an infinite period setting if the discount rate, δ, satisfies:*

$$\delta > \frac{1}{2}$$

Proof. See Appendix 1.4.

To evaluate when collusion is particularly attractive the partial derivatives of $E\left(PP^{ColCol}\right) - E\left(PP^{RR}\right)$ with respect to the exogenous parameters are included in Appendix 1.4. As expected, as the size of the negative externality, c, increases, the per period benefit of colluding also increases. Also, as the correlation coefficient, ρ, increases, the benefits of collusion become larger. This is because, as ρ increases and RPE becomes more effective at removing risk, additional effort is induced. Hence, the magnitude of the negative effort externalities increase as ρ increases. Similarly, as $R\sigma^2$ decreases, the benefits of collusion increase because each principal, acting independently, would induce additional effort as $R\sigma^2$ falls.

1.5.2. Collusion to not use RPE

Whilst in the current model, colluding to use APE instead of RPE does not seem particularly attractive it might be an attractive form of collusion in practice. Firstly, colluding to not use APE is a simpler decision than co-ordinating contract weight collusion since the former simply involves a binary decision. Secondly, whether RPE or APE is used falls within the topics covered in the remuneration section of annual reports. This observability would increase firms' capacity to identify true defections and punish them. The fact that company reports are legal documents increases their reliability as information sources further. In contrast, detailed information about specific incentive contracts, i.e. the actual contract weights used, is rarely, if ever, publicly revealed. As a result, it also seems sensible to derive the critical discount factor which makes collusion to use APE instead of RPE sustainable.

Proposition 1.3 *In an infinitely repeated setting* (APE, APE) *can be supported as a subgame perfect Nash equilibrium if the discount factor,* δ*, satisfies:*

$$\delta > \frac{b}{2c}$$

Proof. See Appendix 1.4.

From the starting assumption $b > c$, the minimum discount factor to collude to not use RPE is always higher than the minimum discount factor to sustain contract weight collusion.[31] This result is natural as collusion to not use RPE only partially internalises the effort externality. This also explains why, in Proposition 1.3, δ depends on the relative magnitudes of b and c. As b becomes smaller relative to c, the negative effort externality becomes larger relative to the marginal return from own-agent effort. Hence collusion to restrict effort becomes more attractive and is easier to sustain.

1.5.3. A role for compensation consultants?

As already noted, publicly available pay information is probably too limited to allow contract weight collusion to be co-ordinated. If firms wanted to exchange more precise information[32], an obvious conduit would be via compensation consultants. A number of articles have been published investigating the role of compensation consultants.[33] However, these articles focus on consultants' conflicts of interest when making pay recommendations and whether they increase pay levels. That compensation consultants might help support product market collusion is novel.

Whilst no legal cases involving incentive contracts and product market collusion have been found, some cases have alleged collusion between firms to gain monopsony power in the labour market. Indeed, in *Todd v. Exxon Corp.*[34] it was alleged that a large compensation consultancy facilitated the exchange of salary information

[31] The condition $c < b < 2c$ for a prisoners' dilemma to occur ensures $\delta < 1$ in Proposition 1.3.
[32] The traditional view is that firms operate in a market for "talent" and fight to retain high-quality staff. In this context, details of compensation packages could be a competitive advantage and so would be guarded closely.
[33] For example, see Murphy and Sandino (2010), Goh and Gupta (2010) and Conyon (2011).
[34] See *Todd v. Exxon Corp.* 275 F.3d 191 (2d Cir. 2001)

amongst oil and petrochemical companies. The ruling in this case, as described by Skonberg et al (2006), found the detailed discussion of compensation survey data in meetings to be "troubling". Importantly, the ruling confirmed that colluding in incentive contracts is covered by antitrust law.[35] Similarly, Miles (2007) reports that nurses brought cases against hospitals in several US cities[36] alleging a similar sharing of information via surveys and discussions at recruitment fairs. There are a variety of potential reasons why cases linking incentive contracts to product market collusion have not been found. It could be because firms have never used contracts in this way, or because other, clearer, evidence of collusion is available to prosecutors, or because antitrust authorities lack awareness regarding this issue.

1.5.4. Empirical evidence

Due to the limited number of legal cases, most evidence regarding an information exchange role for compensation consultants is circumstantial. Compensation consultants certainly have access to large amounts of private contractual information. In particular, the Towers Watson (Towers Perrin) surveys of executive pay are widely quoted in the literature.

Another way to assess the plausibility of information exchange is to investigate whether individual consulting firms are employed by firms that have an incentive to

[35] "FTC Investigates Oil Firms Over Hiring, Wages", The Wall Street Journal, 26 April 2010, reported that *Todd v. Exxon Corp.* was settled out of court in 2009 along with a series of similar cases. Exxon did not admit liability or wrongdoing in the settement.

[36] For example, see, Third Amended Complaint, *Reed v. Advocate Health Care*, No. 6C 3337 (N.D. III filed Feb. 27 2007)

collude. In the UK, USA and Canada the consulting industry does appear fairly concentrated. Conyon (2011) reports that, in 2010, in the US the 5 leading consultancies were employed by 75% of S&P 500 firms, 70% of S&P 1500 firms and 60% of Russell 3000 firms. In the UK, Goh and Gupta (2010) report that amongst FTSE 350 firms the top two consultancy firms had a 48% market share. Future work could take data from company reports to look in more detail at the firms which employ particular compensation consultants.

If the firms in a particular industry were found to use a particular compensation consultant, it remains a long way from proving collusion. There are legitimate reasons why consultancies might specialise in particular industries, such as sector-specific knowledge to enable benchmarking. Nevertheless, there is value in conducting the suggested analysis. If a compensation consultant's clients were spread randomly across the economy, it would suggest limited potential for information exchange. Also, one could look at the distribution of consultancies across firms that have been investigated for product market collusion.

One issue with the data available is that it focuses on the setting of pay for senior executives. A key question is the role compensation consultancies play in determining incentives below board level. To support product market collusion it is likely that incentive contracts at lower organisational levels would also need to be altered. Indeed, the actual decision to collude may be taken by managers at lower organisational levels.

The other potential evidence regarding incentive contract collusion relates to industry variations in RPE usage. As noted in the introduction, there is considerable variation in RPE usage by industry. There are a range of explanations for this, but the desirability of collusion to limit agent effort is also likely to vary by industry and colluding to not use RPE may be a practical way to limit agent effort. However, studies investigating the link between competition, proxied by industry concentration measures, and RPE usage find mixed results. Joh (1999), using Japanese data and measuring RPE implicitly, finds that, in less concentrated industries, executive compensation has a positive relationship with industry profits of a greater magnitude than in more concentrated industries. Joh argues this is consistent with the benefits of collusion being higher in less concentrated industries. In contrast, using explicit RPE data from the US, Gong et al (2010) find that less concentrated industries are more likely to use RPE with a negative weight on rival-firm profits. Lastly, both De Angelis and Grinstein (2010) and Carter et al (2009) find no significant relationship between concentration and RPE.

1.5.5. Other comments on incentive contract collusion

Only collusion between principals is considered at present. Agents themselves could choose to collude regarding their effort choices. Indeed, a principal's incentive contract choice could influence the attractiveness/feasibility of agents colluding. This type of collusion between agents, or agents and their supervisors, in a single firm has been addressed by the principal-agent literature.[37] However, collusion in the present

[37] For example, see Tirole (1986) and Itoh (1992).

setting is slightly different. Whilst agents colluding themselves might try to game the incentive schemes offered, it is possible that such collusion could still benefit the principals, as it may also internalise the effort externality.[38] A full exploration of this issue is left for future work.

Once one considers the risk of antitrust action, other interesting issues emerge. Whilst there is no difference between the owners or managers colluding, in terms of fines facing firms, for an owner as an individual there may be a difference. If those who organise collusion are criminally liable as individuals, firm owners may wish to avoid colluding directly when setting incentive contracts. Instead, owners may wish to write incentive contracts independently, but in such a way that managers are encouraged to initiate collusion. For example, it would be interesting to investigate the intuition of Lundgren (1996) that RPE is less likely to encourage product market collusion than APE.

1.6. Positive Effort Externalities

When there are positive effort externalities the solution method is identical to that when there are negative effort externalities. The only change is to the profit function.

$$\pi_i(e_i, e_j, \varepsilon_i) = F + be_i + de_j + \varepsilon_i$$

is now used. Given the similarity, only the key results for the positive externality case are stated below. Additional detail is provided in Appendix 1.5.

[38] Whilst in Spagnolo (2000, 2005) managers take the decision to collude, these models do not incorporate unobservable effort. As such, there is no potential for the agents to game an incentive scheme itself.

When APE is used, $\alpha^A_+ = \alpha^A$ and $e^A_+ = e^A$, i.e. the optimal incentive and effort levels from the negative externality case remain optimal.[39] This result occurs because the externality's direction does not affect the marginal return to own agent effort or the variance of π_i. However, since the positive externality alters $E(\pi_i)$, the agent's base wage does need to change to ensure the agent's participation constraint continues to bind with equality.

When RPE is used, the optimal incentive weights and effort levels change relative to the negative externality case. This is because e_i now enters into firm j's profit function with a positive sign. As such, a trade-off exists when determining the value, and sign, of β^R_+. To reduce the variance of the agent's transfer payment requires $\beta^R_+ < 0$. However, as e_i enters positively into π_j, to harness π_j as a direct signal of e_i requires $\beta^R_+ > 0$. If $\beta^R_+ > 0$ the variance of T_i increases compared to APE, implying a higher risk premium must be paid.[40][41]

When RPE is used, the optimal weight to place on own firm profits is:

$$\alpha^R_+ = \frac{b\,(b - d\rho)}{2R\sigma^2(1-\rho^2) + b^2 + d^2 - 2bd\rho}$$

and the optimal weight to place on rival firm profits is:

$$\beta^R_+ = \frac{b\,(d - b\rho)}{b^2 + d^2 - 2bd\rho + 2R\sigma^2(1-\rho^2)}$$

[39]This result is demonstrated in Appendix 1.5. The subscript "+" denotes that α^A_+ and e^A_+ refer to the positive externality case.

[40]Both Itoh (1992) and Choi (1993) analyse this trade-off for the case of a single principal incentivising two agents.

[41]When there is a negative externality, e_i enters into π_j as a negative term. As such, $\beta^R < 0$ both minimises risk and maximises incentive strength.

The optimal effort level chosen by each agent is:

$$e_+^R = \frac{b\left(b^2 + d^2 - 2bd\rho\right)}{2\left(b^2 + d^2 - 2bd\rho + 2R\sigma^2(1-\rho^2)\right)}$$

Lemma 1.3 *α_+^R is always positive. When $b > \frac{d}{\rho}$, β_+^R is negative and, when $b < \frac{d}{\rho}$, β_+^R is positive. When $d = b\rho$, β_+^R equals zero and it is optimal to use APE.*

Proof. From $b > d$ and $\rho \in [0,1]$, $b > d\rho$ must hold ensuring the numerator of α_+^R is always positive. To confirm that the denominator is also positive see Appendix 1.5 and the proof that $E\left(PP_i\right)$ following (RPE, RPE) is concave. Hence, $\alpha_+^R > 0$. The same starting assumptions explain why β_+^R can be positive, negative or zero.[42] ■

As in the negative externalities case, effort is always higher when RPE is used. When there are positive effort externalities and $\rho \in (0,1)$,[43] the number of comparative statics where the sign depends on the value of exogenous parameters is higher than in the negative externalities case. Notably, even the comparative statics for e_+^R can have varying signs. For example, $\frac{\partial e_+^R}{\partial \rho}$ can be positive or negative. When b is relatively large, $\frac{\partial e_+^R}{\partial \rho} > 0$, and when d is relatively large, $\frac{\partial e_+^R}{\partial \rho} < 0$. This makes sense since, when b is large, $\beta_+^R < 0$ and RPE is used for risk reduction purposes. An increase in ρ allows additional risk to be removed by RPE reducing agency costs so additional effort is induced. When d is large, $\beta_+^R > 0$. As such, an increase in ρ leads

[42]When $\rho \in [-1,0)$, β_+^R is guaranteed to be positive. When the correlation coefficient is negative, $\beta_+^R > 0$ allows rival firm profits to be used both for risk reduction and as an informative signal of own agent effort.

[43]A discussion of the comparative statics both when $\rho \in [0,1]$ and when $\rho \in [-1,0)$ is provided in Appendix 1.6.

to the transfer payment variance increasing which means agency costs increase and less effort is induced.

The principals' expected pay-offs following each stage 1 action pair are shown in Figure 1.3 in Appendix 1.5. The expressions for the optimal base wages are also shown in Appendix 1.5.

Proposition 1.4 *(APE, APE) does not offer a Pareto improvement over (RPE, RPE).*

Proof. (APE, APE) does not offer a Pareto improvement over (RPE, RPE) if $E\left(PP_{+}^{RR}\right) > E\left(PP_{+}^{AA}\right)$. This condition requires:

$$E\left(PP_{+}^{RR}\right) - E\left(PP_{+}^{AA}\right) = \frac{Rb\sigma^2\left(b+2d\right)\left(b\rho-d\right)^2}{2\left(2R\sigma^2+b^2\right)\left(2R\sigma^2\left(1-\rho^2\right)+b^2+d^2-2bd\rho\right)} > 0,$$

which always holds. ■

Proposition 1.4 shows the principals do not have an incentive to collude to stop the use of RPE, when positive effort externalities exist. The reason APE never offers increased pay-offs relative to RPE is because using RPE always increases agents' effort. In the case of positive effort externalities, this moves the principals' pay-offs towards those achievable via co-ordination. Indeed, the incentive for principals to collude when setting the contract weights remains. Investigating collusion in the presence of positive effort externalities is left for future work.[44] However, it is worth noting, that

[44] Although, any results from the collusion/co-ordination case are likely to be similar to the work by Itoh (1992) and Choi (1993).

the ability to co-ordinate contracting decisions to internalise effort externalities could be an important factor encouraging mergers and joint ventures.[45]

1.6.1. Official encouragement to use RPE

In the UK, Liu and Stark (2009) note, that the quasi-official encouragement of RPE began with the Greenbury (1995) report. It continued with the Association of British Insurers (ABI) (1996, 1999) guidelines for remuneration and the Financial Reporting Council's Combined Code (2008). In the US, De Angelis and Grinstein (2010) highlight that, from 2006, a narrative discussion of compensation schemes and the criteria used to determine payouts has been required in public firms' proxy statements.[46] Following the principal-agent literature, it is RPE where a negative weight is placed on rival firms' profits that has been encouraged. The current model implies that if $\beta_{+}^{R} > 0$ is profit-maximising, acting on this encouragement would be harmful to shareholders. Hence, the model shows why leaving the final decision on RPE to companies, rather than legally requiring its use, is advantageous. Hopefully, if a particular form of RPE reduces profits, rational boards will choose not to use it.

Given this result for the case of positive externalities, and the context dependent nature of the comparative statics, it is worth commenting further on the encouragement to use RPE across firms. Firstly, in the original theoretical work on RPE, such

[45] In the organisational design literature the benefits of co-ordination, and the trade-offs it potentially entails, receive considerable attention. For example, see Alonso et al (2008a, 2008b) and Dessein et al (2010). However, these three papers are rather different to the present chapter. They interpret co-ordination as the similarity of decisions rather than internalising externalities, do not consider RPE, and emphasise hidden information along with strategic communication.

[46] For a brief overview see pages 28-29 of SEC Release No. 33-8732A.

as Holmstrom (1982), references to RPE across firms were largely in passing, suggesting the theoretical underpinning for its benefits may have been overplayed. Secondly, regarding RPE encouragement in the UK, it is noticeable that RPE receives much less emphasis in the ABI's most recent "Principles of Remuneration" published in 2011. Whilst this could reflect the more recent theoretical results regarding RPE, it is more likely to reflect changes in the public debate on executive compensation. Post-2008, the focus has shifted from avoiding "rewards for luck" to avoiding "rewards for failure" and addressing the short-term nature of many pay awards.

Such shifts reflect changing circumstances and suggest that official responses to executive compensation, whilst often phrased in the language of supporting shareholders' interests, actually reflect the interplay of many different forces.[47] The encouragement of RPE probably cannot be divorced from the political need for executive compensation to appear "deserved".

Thirdly, the model's relatively complex results regarding the optimal design of RPE schemes occur when attention is restricted to linear contracts. Real incentive schemes for senior executives contain far greater complexity than the contracts analysed in the theoretical literature. The schemes used often contain multiple elements, covering many different activities and are designed to cover performance over different time periods.[48] A legitimate question to ask is whether the incentives offered by such packages, when taken as a whole, are understood sufficiently to allow robust policy recommendations.

[47]See Murphy (2011, 2012).
[48]See Murphy (1999, 2011, 2012).

In defence of the authorities, the reduced form profit functions analysed in this chapter do not allow full welfare comparisons to be made. As such, whilst RPE may not always increase firms' profits, it could be that the extra effort it induces does raise total welfare. If this were the case it would represent an interesting conflict between profit-maximising and welfare-maximising objectives when incentive contracts are written.

1.7. Conclusion

This paper considers RPE across firms when effort exerted in one firm imposes externalities on another firms' profits. Whilst the model analysed is simple, it does emphasise that, if externalities are considered, the relatively straightforward relationships highlighted by the early RPE literature may no longer apply. As such, constructing an "optimal" incentive contract may not be intuitive, even in simple settings.

Furthermore, the paper shows that, even when it is individually rational for firms to use RPE, sufficiently large negative effort externalities can mean industry profits are increased by firms colluding to not use RPE. The negative effort externalities make it beneficial to limit the effort exerted by agents, and colluding not to use RPE is a crude, but straightforward, way to do this. Firms can increase their profits still further by colluding when setting the contract weights, α and β. Notably, observing RPE with a negative weight on rival-firm profits does not rule out the possibility that collusion in contract weights is occurring.

Once one recognises the potential for firms to collude via incentive contracts, it becomes interesting to consider incentive contracting decisions from the perspective of cartel theory. In particular, it seems interesting to ask whether compensation consultants can be used as information exchange devices to facilitate incentive contract collusion. That this type of collusion could affect product market outcomes is novel and warrants further investigation.

In conclusion, this paper highlights the importance, and potential difficulty, of modelling firms' contracting decisions when the performance measures used include the performance of other firms. In such settings, the paper indicates that caution should be used before particular incentives are widely encouraged, and that incentive contracts may offer an additional mechanism through which firms can collude.

1.8. Appendices

1.8.1. Appendix 1.1 - Additional Material for Sections 1.3 and 1.4

Equivalence of Maximising EU_i and CE_i

The following proof is taken, with changed notation, from Bolton and Dewatripont (2005).

Lemma A1.1 *For the utility function described in Section 1.3, the agent's expected utility maximisation problem reduces to a maximisation problem regarding the agent's certainty equivalent.*

Proof Assume the agent has constant absolute risk-aversion (CARA) preferences represented by the utility function:

$$u\left(T_i, e_i\right) = -e^{-R(T_i - g(e_i))}$$

Consider the case of APE and a negative effort externality. Inserting the agent's transfer payment given by (1.4), the profit function given by (1.1) and the expression for $g\left(e_i\right)$ into $u\left(T_i, e_i\right)$ gives agent i's maximisation problem as:

$$\max_{e_i} EU_i = E\left[-e^{-R\left(w_i+\alpha_i(F+be_i-ce_j+\varepsilon_i)-e_i^2\right)}\right] = -e^{-R\left(w_i+\alpha_i(F+be_i-ce_j)-e_i^2\right)}E\left[e^{-R\alpha_i\varepsilon_i}\right]$$

When $\varepsilon_i \sim N(0, \sigma_i^2)$, and letting π represent pi:

$$E\left[e^{-R\alpha_i\varepsilon_i}\right] = \frac{1}{\sqrt{2\pi\sigma}}\int e^{-R\alpha_i\varepsilon_i}e^{-\frac{\varepsilon_i^2}{2\sigma^2}}d\varepsilon_i = \frac{1}{\sqrt{2\pi\sigma}}\int e^{-\frac{\left(\varepsilon_i^2+2R\alpha_i\varepsilon_i\sigma^2\right)}{2\sigma^2}}d\varepsilon_i$$

$$= \frac{1}{\sqrt{2\pi\sigma}} \int e^{-\frac{\left(\varepsilon_i + R\alpha_i\sigma^2\right)^2 - R^2\alpha_i^2\sigma^4}{2\sigma^2}} d\varepsilon_i = e^{\frac{1}{2}R^2\alpha_i^2\sigma^2} \frac{1}{\sqrt{2\pi\sigma}} \int e^{-\frac{\left(\varepsilon_i + R\alpha_i\sigma^2\right)^2}{2\sigma^2}} d\varepsilon_i$$

Noting that:

$$\frac{1}{\sqrt{2\pi\sigma}} \int e^{-\frac{\left(\varepsilon_i + R\alpha_i\sigma^2\right)^2}{2\sigma^2}} d\varepsilon_i = 1$$

as the LHS represents a normal distribution with mean $-R\alpha_i\sigma^2$ and variance σ^2, it must be the case that:

$$E\left[e^{-R\alpha_i\varepsilon_i}\right] = e^{\frac{1}{2}R^2\alpha_i^2\sigma_i^2}$$

Hence:

$$EU_i = -e^{-R\left(w_i + \alpha_i(F + be_i - ce_j) - e_i^2\right)} e^{\frac{1}{2}R^2\alpha_i^2\sigma_i^2} = -e^{-R\left(w_i + \alpha_i(F + be_i - ce_j) - e_i^2 - \frac{1}{2}R\alpha_i^2\sigma_i^2\right)}$$

where the term inside brackets is agent i's certainty equivalent,

$$CE_i = w_i + \alpha_i\left(F + be_i - ce_j\right) - e_i^2 - \frac{1}{2}R\alpha_i^2\sigma_i^2$$

Since the utility function $U(y) = -e^{-Ry}$ is increasing concave in y, maximising the value of CE_i maximises EU_i. ■

Using RPE does not change the equivalence result, merely the function representing the certainty equivalent. When RPE is used:

$$\begin{aligned} EU_i &= E\left[-e^{-R\left(w_i + \alpha_i(F + be_i - ce_j + \varepsilon_i) + \beta_i(F + be_j - ce_i + \varepsilon_j) - e_i^2\right)}\right] \\ &= -e^{-R\left(w_i + \alpha_i(F + be_i - ce_j + \varepsilon_i) + \beta_i(F + be_j - ce_i + \varepsilon_j) - e_i^2\right)} E\left[-e^{-R(\alpha_i\varepsilon_i + \beta_i\varepsilon_j)}\right] \end{aligned}$$

As ε_i and ε_j are correlated normal random variables:

$$E\left[-e^{-R(\alpha_i\varepsilon_i+\beta_i\varepsilon_j)}\right]=-e^{-\frac{1}{2}R^2\left(\alpha_i^2\sigma_i^2+\beta_i^2\sigma_j^2+2\rho\alpha_i\beta_i\sigma_i\sigma_j\right)}$$

and, as σ_i^2 and σ_j^2 share the same magnitude, this expression reduces to:[49]

$$-e^{-\frac{1}{2}R^2\sigma^2\left(\alpha_i^2+2\rho\alpha_i\beta_i+\beta_i^2\right)}$$

Hence, agent i's certainty equivalent is:

$$CE_i=w_i+\alpha_i\left(F+be_i-e_j\right)+\beta_i\left(F+be_j-e_i\right)$$

$$-e_i^2-\frac{R}{2}\sigma^2\left(\alpha_i^2+2\rho\alpha_i\beta_i+\beta_i^2\right)$$

Proof $E\left(PP_i\right)$ **is Concave Following** (RPE, RPE)

Following (RPE, RPE) principal i's expected pay-off is:

$$E\left(PP_i\right)=F+\frac{1}{2}\left(b^2\alpha_i+c^2\beta_j-bc\alpha_j-bc\beta_i\right)-\widehat{u}-\left(\frac{1}{2}\left(\alpha_ib-\beta_ic\right)\right)^2$$

$$-\frac{R}{2}\sigma^2\left(\alpha_i^2+2\rho\alpha_i\beta_i+\beta_i^2\right)$$

For this to be concave in α_i and β_i both of the following conditions must hold:

(i) $\frac{\partial^2E(PP_i)}{\partial\alpha_i^2}<0$ and

(ii) $\left(\frac{\partial^2E(PP_i)}{\partial\alpha_i^2}\right)\left(\frac{\partial^2E(PP_i)}{\partial\beta_i^2}\right)-\left(\frac{\partial^2E(PP_i)}{\partial\alpha_i\partial\beta_i}\right)^2>0$

[49]In general, for two correlated random variables $X_1\sim N\left(0,\sigma_1^2\right)$ and $X_2\sim N\left(0,\sigma_2^2\right)$ where the correlation co-efficient is ρ:

$$\begin{aligned}Var\left(\alpha X_1,\beta X_2\right)&=\alpha^2Var\left(X_1\right)+\beta^2Var\left(X_2\right)+\alpha\beta Cov\left(X_1,X_2\right)\\&=\alpha^2\sigma_1^2+\beta^2\sigma_2^2+2\alpha\beta\rho\sigma_1\sigma_2\end{aligned}$$

Assuming the variances share the same magnitude gives:

$$Var\left(\alpha X_1,\beta X_2\right)=\left(\alpha^2+\beta^2+2\alpha\beta\rho\right)\sigma^2$$

$\frac{\partial^2 E(PP_i)}{\partial \alpha_i^2}$ is given by:

$$\frac{\partial^2 E\left(PP_i\right)}{\partial \alpha_i^2} = -\left(\frac{1}{2}b^2 + R\sigma^2\right)$$

which is always negative. The starting assumption $\rho \in [0,1]$ ensures that (ii):

$$\left(\frac{\partial^2 E\left(PP_i\right)}{\partial \alpha_i^2}\right)\left(\frac{\partial^2 E\left(PP_i\right)}{\partial \beta_i^2}\right) - \left(\frac{\partial^2 E\left(PP_i\right)}{\partial \alpha_i \partial \beta_i}\right)^2 = \frac{1}{2}R\sigma^2\left(2R\sigma^2\left(1-\rho^2\right) + b^2 + c^2 + 2bc\rho\right)$$

is always positive. As (i) and (ii) are both satisfied $E\left(PP_i\right)$ is concave. ■

Solutions Following Stage 1 Action Pairs Other Than (RPE, RPE)

Following (APE, APE)

Imposing $\beta_i = 0$, agent i's maximisation problem in stage 3 reduces to:

$$\max_{e_i} CE_i = w_i + \alpha_i\left(F + be_i - ce_j\right) - e_i^2 - \frac{R}{2}\alpha_i^2\sigma^2 \tag{1.11}$$

This problem is concave in e_i. Hence, setting $\frac{\partial CE_i}{\partial e_i} = 0$ gives agent i's optimal effort as:

$$e_i = \frac{1}{2}b\alpha_i \tag{1.12}$$

By symmetry, agent j's optimal effort is $e_j = \frac{1}{2}b\alpha_j$.

In the stage 2 subgame principal i maximises $E(PP_i)$ subject to the ICC and PC. Imposing $\beta_i = 0$ and $\beta_j = 0$, the unconstrained maximisation problem in (1.10) reduces to:

$$\max_{\alpha_i} E(PP_i) = F + \frac{1}{2}b\left(b\alpha_i - c\alpha_j\right) - \widehat{u} - \left(\frac{1}{2}b\alpha_i\right)^2 - \frac{R}{2}\alpha_i^2\sigma^2$$

By inspection, this problem is concave. Solving the resulting FOC gives the optimal value of α_i under APE as:

$$\alpha^A = \frac{b^2}{b^2 + 2R\sigma^2}$$

Inserting α^A back into (1.12) gives the optimal effort level as:

$$e^A = \frac{b^3}{2\left(b^2 + 2R\sigma^2\right)}$$

By symmetry, α^A and e^A also give the optimal values for principal-agent pair j. The optimal wage w^{AA} is shown in a later sub-section of this appendix.

Following (RPE, APE) **and** (APE, RPE)

As already noted, the optimal choices of α_i, β_i and e_i are invariant to the optimal choices of principal-agent pair j and so are not affected by principal j's decision between APE and RPE. As such, the values of α^A, e^A, α^R, β^R and e^R can be taken from sections 1.4.1 and 1.4.2.

However, the action pair selected in stage 1 does alter the profits achieved by each firm. To ensure each agent's participation constraint continues to hold with equality, the base wage, w_i, must change after each stage 1 action pair. The values of w^{RA} and w^{AR} are shown in the following sub-section.

Following (RPE, APE), the expected pay-off for principal i, expressed in terms of the contract weights, is given by (1.10) with the restriction $\beta_j = 0$ imposed. To find $E\left(PP^{RA}\right)$ the values $\alpha_i = \alpha^R$, $\beta_i = \beta^R$ and $\alpha_j = \alpha^A$ are then inserted. Following (APE, RPE), the principal's expected pay-off is again given by (1.10) but now with the restriction $\beta_i = 0$ imposed. To find $E\left(PP^{AR}\right)$, the values $\alpha_i = \alpha^A$, $\alpha_j = \alpha^R$ and $\beta_j = \beta^R$ are then inserted. By symmetry, $E\left(PP^{RA}\right)$ also represents principal j's expected pay-off following (APE, RPE), and $E\left(PP^{AR}\right)$ also represents principal j's expected pay-off following (RPE, APE).

Expressions for w - Negative Effort Externality

Following (APE, APE) the optimal base wage is:

$$w^{AA} = \widehat{u} - \left(\frac{b^2}{b^2 + 2R\sigma^2}\right)\left[F + \frac{1}{4}\left(\frac{b^2}{b^2 + 2R\sigma^2}\right)\left(b\left(b - 2c\right) - 2R\sigma^2\right)\right]$$

Following (RPE, RPE) the optimal base wage is:

$$w^{RR} = \widehat{u} + \frac{b}{2R\sigma^2\rho^2 - 2R\sigma^2 - b^2 - c^2 - 2bc\rho}$$

$$\times \left[\begin{array}{c} (1-\rho)\left(b - c\right)F \\ + \frac{b\left(b^2+c^2+2bc\rho\right)\left(\frac{1}{2}\left(2b^2\rho+2c^2\rho+4bc-b^2-c^2-2bc\rho\right)+R\sigma^2\left(1-\rho^2\right)\right)}{2(2R\sigma^2\rho^2-2R\sigma^2-b^2-c^2-2bc\rho)} \end{array}\right]$$

Following (RPE, APE) the optimal base wage for agent i is:

$$w^{RA} = \widehat{u} - \frac{b\left(b - c\right)\left(1 - \rho\right)F}{2R\sigma^2 + b^2 + c^2 + 2bc\rho - 2R\sigma^2\rho^2}$$

$$-\frac{b}{2\left(2R\sigma^2(1-\rho^2) + b^2 + c^2 + 2bc\rho\right)}$$

$$\times\begin{pmatrix} (b+c\rho)\left(\frac{b^2\left(b^2+2\rho bc+c^2\right)}{2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho}-\frac{cb^3}{b^2+2R\sigma^2}\right) \\ -(c+b\rho)\left(\frac{b^4}{b^2+2R\sigma^2}-\frac{cb\left(b^2+2\rho bc+c^2\right)}{2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho}\right) \\ -\frac{1}{2}\frac{b\left(b^2+2\rho bc+c^2\right)^2}{2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho} \end{pmatrix}$$

$$+\frac{b^2\left(b^2+c^2+2bc\rho\right)(1-\rho)(\rho+1)R\sigma^2}{2\left(-2R\sigma^2-b^2-c^2-2bc\rho+2R\sigma^2\rho^2\right)^2}$$

Following (APE, RPE) the optimal base wage for agent i is:

$$w^{AR}=\widehat{u}+\left(\frac{b^2}{b^2+2R\sigma^2}\right)$$

$$\times\left(\frac{1}{2}R\sigma^2-F+\frac{1}{2}\left(\frac{c\left(b^2+2\rho bc+c^2\right)}{2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho}-\frac{b^3}{2\left(b^2+2R\sigma^2\right)}\right)\right)$$

Stage 1 Pay-off Matrix

Using $E\left(PP^{AA}\right)$, $E(PP^{RR})$, $E\left(PP^{RA}\right)$ and $E(PP^{AR})$ a matrix of expected pay-offs for the principals in the stage 1 subgame can be formed. The Nash equilibrium in stage 1 can then be identified in a straightforward fashion. The pay-off matrix is shown in Figure 1.

Principal i \ Principal j	APE	RPE
APE	$F+\frac{b^2(b-2c)}{4(2R\sigma^2+b^2)}-\hat{u}$, $F+\frac{b^2(b-2c)}{4(2R\sigma^2+b^2)}-\hat{u}$	$F-\frac{b\left(2R\sigma^2\left(2bc(b+2c\rho)+2c^3-b^3(1-\rho^2)\right)-b^2(b-2c)\left(b^2+c^2+2bc\rho\right)\right)}{4(b^2+2R\sigma^2)(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))}-\hat{u}$, $F+\frac{b^2(2R\sigma^2\left((b-c)^2+2bc\rho(1+\rho)\right)+b(b-2c)\left(b^2+c^2+2bc\rho\right))}{4(b^2+2R\sigma^2)(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2)}-\hat{u}$
RPE	$F+\frac{b^2(2R\sigma^2\left((b-c)^2+2bc\rho(1+\rho)\right)+b(b-2c)\left(b^2+c^2+2bc\rho\right))}{4(b^2+2R\sigma^2)(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2)}-\hat{u}$, $F-\frac{b\left(2R\sigma^2\left(2bc(b+2c\rho)+2c^3-b^3(1-\rho^2)\right)-b^2(b-2c)\left(b^2+c^2+2bc\rho\right)\right)}{4(b^2+2R\sigma^2)(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))}-\hat{u}$	$F+\frac{b(b-2c)(b^2+2\rho bc+c^2)}{4(2R\sigma^2(1-\rho^2)+b^2+2\rho bc+c^2)}-\hat{u}$, $F+\frac{b(b-2c)(b^2+2\rho bc+c^2)}{4(2R\sigma^2(1-\rho^2)+b^2+2\rho bc+c^2)}-\hat{u}$

Figure 1.1: The principals' expected pay-offs in stage 1 when there are negative effort externalities.

1.8.2. Appendix 1.2 - When an Algebraic Solution is not Possible

This appendix assumes the reader has read Sections 1.3 and 1.4.

Lemma A1.2 *If the profit function takes the form:*

$$\pi_i = F + be_i - e_j - e_i e_j + \varepsilon_i$$

an algebraic solution is no longer possible following the stage 1 action pair (RPE, RPE)*.*

Proof. Let firm i's profit be:

$$\pi_i = F + be_i - e_j - e_i e_j + \varepsilon_i$$

and assume $b > 1$. The model is otherwise identical to that described in Section 1.3.

Stage 3 subgame:

Following (RPE, RPE) agent i receives the transfer payment:

$$T_i = w_i + \alpha_i (F + be_i - e_j - e_i e_j + \varepsilon_i) + \beta_i (F + be_j - e_i - e_j e_i + \varepsilon_j)$$

The variance of T_i is given by (1.6). Hence, agent i's maximisation problem, holding e_j fixed, is:

$$\max_{e_i} CE_i = w_i + \alpha_i (F + be_i - e_j - e_i e_j) + \beta_i (F + be_j - e_i - e_i e_j)$$

$$-e_i^2 - \frac{R}{2}\sigma^2 \left(\alpha_i^2 + 2\rho\alpha_i\beta_i + \beta_i^2\right)$$

By inspection, this problem is concave in e_i and so the FOC is a sufficient condition for utility maximisation. Agent i's FOC is:

$$\frac{\partial CE_i}{\partial e_i} = \alpha_i b - \alpha_i e_j - \beta_i - \beta_i e_j - 2e_i = 0 \tag{1.13}$$

Compared to the model in Section 1.3 e_j now enters into agent i's FOC. Agent i's optimal effort choice now depends on agent j's effort choice. As such, the Nash equilibrium in the stage 3 subgame is no longer trivial. Re-arranging (1.13), agent i's best response function is:

$$e_i^{BR} = \frac{1}{2}(\alpha_i b - \beta_i) - \frac{1}{2}(\alpha_i + \beta_i) e_j$$

and, by symmetry, agent j's best response function is:

$$e_j^{BR} = \frac{1}{2}\left(\alpha_j b - \beta_j\right) - \frac{1}{2}\left(\alpha_j + \beta_j\right) e_i$$

Define a Nash equilibrium in the stage 3 subgame as the effort choice pair $\left(e_i^*, e_j^*\right)$ such that the following two equations are satisfied:

$$e_i^* = \frac{1}{2}(\alpha_i b - \beta_i) - \frac{1}{2}(\alpha_i + \beta_i) e_j^* \tag{1.14}$$

$$e_j^* = \frac{1}{2}\left(\alpha_j b - \beta_j\right) - \frac{1}{2}\left(\alpha_j + \beta_j\right) e_i^* \tag{1.15}$$

Assume unique optimal values for α_i, β_i, α_j and β_j exist. Given this assumption, conditions can be stated guaranteeing the existence of a Nash equilibrium in the stage 3 subgame. Note that e_i^{BR} and e_j^{BR} are continuous and linear in e_j and e_i

respectively. The starting assumption $e_i, e_j \in [0, \overline{e}]$ means that (e_i, e_j)-space is closed and bounded. If the best response functions cross, the point where they do will be a Nash equilibrium. As the equations are linear, there are only three possible outcomes: the lines do not cross in the portion of (e_i, e_j)-space considered; they cross once giving a unique Nash equilibrium; or they coincide giving an infinite number of solutions. A Nash equilibrium in this subgame is guaranteed to exist if $\alpha_i b > \beta_i$, $\alpha_j b > \beta_j$, $\alpha_i + \beta_i > 0$ and $\alpha_j + \beta_j > 0$. If these conditions hold the resulting equilibrium will be unique unless $\alpha_i^* = \alpha_j^*$, $\beta_i^* = \beta_j^*$ and $\alpha_i^* + \beta_i^* = 2$.[50]

For now, assume an equilibrium in stage 3 exists. Solving (1.14) and (1.15) as a pair of simultaneous equations gives the equilibrium effort levels as:

$$e_i^* = \frac{\left(b\alpha_j - \beta_j\right)\left(\alpha_i + \beta_i\right) - 2\left(b\alpha_i - \beta_i\right)}{\left(\alpha_j + \beta_j\right)\left(\alpha_i + \beta_i\right) - 4} \tag{1.16}$$

and

$$e_j^* = \frac{\left(b\alpha_i - \beta_i\right)\left(\alpha_j + \beta_j\right) - 2\left(b\alpha_j - \beta_j\right)}{\left(\alpha_j + \beta_j\right)\left(\alpha_i + \beta_i\right) - 4} \tag{1.17}$$

Stage 2 subgame:

Principal i's unconstrained maximisation problem is:

$$\max_{\alpha_i, \beta_i} E(PP_i) = F + be_i^* - e_j^* - e_i^* e_j^* - \left(e_i^*\right)^2 - \widehat{u} - \frac{R}{2}\sigma^2\left(\alpha_i^2 + 2\rho\alpha_i\beta_i + \beta_i^2\right)$$

[50] If these three additional conditions hold e_i^{BR} and e_j^{BR} will coincide and there will be infinitely many solutions. Whilst these conditions are sufficient to prove existence in the situation described, they are not necessary.

where $e_i^*$and e_j^* are given by (1.16) and (1.17). Since the contract weights are not additively separable in $E(PP_i)$, principal i's optimal contract weights will depend on principal j's choice of contract weights. Assuming $E(PP_i)$ and $E(PP_j)$ are concave and that an equilibrium exists, the equilibrium contract weights can be found by solving $\frac{\partial E(PP_i)}{\partial \alpha_i} = 0$, $\frac{\partial E(PP_j)}{\partial \alpha_j} = 0$, $\frac{\partial E(PP_i)}{\partial \beta_i} = 0$ and $\frac{\partial E(PP_j)}{\partial \beta_j} = 0$ simultaneously. Assuming a symmetric equilibrium such that $\alpha_i^* = \alpha_j^* = \alpha$ and $\beta_i^* = \beta_j^* = \beta$ the four FOCs can be reduced to the following pair of simultaneous equations:

$$\frac{\partial E(PP_i)}{\partial \alpha_i} = \frac{\partial E(PP_j)}{\partial \alpha_j} =$$

$$\frac{(2b+\beta+b\beta)(2-\alpha-\beta)(4b-2\alpha(2b-1)+2\beta(b+4)+\alpha(\alpha+\beta)(b+1))}{(\alpha+\beta-2)^2(\alpha+\beta+2)^3}$$

$$-\frac{1}{2}R\sigma^2(2\alpha+2\rho\beta) = 0$$

$$\frac{\partial E(PP_i)}{\partial \beta_i} = \frac{\partial E(PP_j)}{\partial \beta_j} =$$

$$-\frac{(\alpha+b\alpha+2)(2-\alpha-\beta)(4b+2\beta(b+4)-2\alpha(2b-1)+\alpha(\alpha+\beta)(b+1))}{(\alpha+\beta-2)^2(\alpha+\beta+2)^3}$$

$$-\frac{1}{2}R\sigma^2(2\beta+2\rho\alpha) = 0$$

These conditions represent sixth-order polynomials in α and β. Hence, an algebraic solution in the stage 2 subgame, and therefore an algebraic solution in the entire game, is not possible. ■

This shows that even if only linear contracts are considered, using RPE can cause principals' maximisation problems to be highly complex. Given the difficulty of obtaining algebraic solutions in a highly stylised model[51], it perhaps questions the ability to design "optimal" RPE contracts in practice.

1.8.3. Appendix1.3 - Asymmetric Profit Functions

Two obvious forms of asymmetry could be introduced: asymmetry regarding firms' profit functions and asymmetry regarding agents' characteristics. Here the former is considered. Let the values of b and c vary by firm, i.e. let:

$$\pi_i = F + b_i e_i - c_i e_j + \varepsilon_i$$

This type of asymmetry does not change the functional form of the expressions for α^A, α^R, β^R, e^A and e^R. From the perspective of principal-agent pair i, all that changes in these expressions is that b becomes b_i and c becomes c_j. For example, α^R becomes:

$$\alpha^R = \frac{b_i\left(b_i + c_j\rho\right)}{2R\sigma^2(1-\rho^2) + b_i^2 + c_j^2 + 2b_i c_j \rho}$$

That the functional form does not change follows from the additive separability of e_i and e_j in $E\left(\pi_i\right)$.[52] Again, as in section 1.4, this property means that, acting independently, each principal will always use RPE. There is no strategic value in unilaterally

[51] A model was also considered where agent i's effort reduced firm i's marginal cost. Again, in stage 2, when RPE was used, the polynomials' orders were too high to allow algebraic solutions.

[52] When principal i and agent i solve their maximisation problems, c_i and b_j do not appear in the resulting FOCs.

committing to use APE. Hence in the stage 1 subgame the Nash equilibrium is again (RPE, RPE).

Introducing asymmetric profit functions, however, does mean the expressions for the principals' expected pay-offs become considerably larger. To keep the expected pay-off functions manageable assume $\rho = 1$. Also assume $b_i > c_i$ and $b_j > c_j$. When $\rho = 1$ and principal i uses APE, the optimal value of α_i is:

$$\alpha_i^A = \frac{b_i^2}{b_i^2 + 2R\sigma^2},$$

agent i's optimal effort is:

$$e_i^A = \frac{b_i^3}{2\left(b_i^2 + 2R\sigma^2\right)},$$

and principal i's expected pay-off is:

$$E\left(PP_i^A\right) = F + b_i e_i - c_i e_j - \widehat{u} - e_i^2 - \frac{R}{2}\alpha_i^2\sigma^2 \tag{1.18}$$

If principal i uses RPE, the optimal contract weights are:

$$\alpha_i^R = \tfrac{b_i}{b_i + c_j} \text{ and } \beta_i^R = -\tfrac{b_i}{b_i + c_j},$$

agent i's optimal effort is:

$$e_i^R = \frac{1}{2}b_i,$$

and principal i's expected pay-off is:

$$E\left(PP_i^R\right) = F + b_i e_i - c_i e_j - \widehat{u} - e_i^2 - \frac{R}{2}\sigma^2\left(\alpha_i^2 + 2\rho\alpha_i\beta_i + \beta_i^2\right) \tag{1.19}$$

Inserting α_i^A, e_i^A, α_i^R, β_i^R, e_j^R, α_j^A, e_j^A, α_j^R, β_j^R and e_j^R into (1.18) and (1.19) gives the pay-off matrix in Figure 1.2. The values of $E\left(PP_i^{RA}\right)$ and $E\left(PP_i^{AR}\right)$ are found by the process described in Appendix 1.1.

Whilst (RPE, RPE) remains the Nash equilibrium in stage 1, asymmetric profit functions do alter the condition for (APE, APE) to offer a Pareto improvement over (RPE, RPE).

Proposition A1.1 *For (APE, APE) to represent a Pareto improvement over (RPE, RPE) both:*

$$\frac{b_i^2\left(b_j^2+2R\sigma^2\right)}{2b_j\left(b_i^2+2R\sigma^2\right)} < c_i < b_i \text{ and } \frac{b_j^2\left(b_i^2+2R\sigma^2\right)}{2b_i\left(b_j^2+2R\sigma^2\right)} < c_j < b_j$$

must hold. If b_i and b_j are related, such that $b_j = \gamma b_i$, both of these conditions can hold simultaneously if $\gamma \in \left[\frac{1}{2}, 2\right]$.

Proof. For (APE, APE) to represent a Pareto improvement over (RPE, RPE) both $E\left(PP_i^{AA}\right) > E\left(PP_i^{RR}\right)$ and $E\left(PP_j^{AA}\right) > E\left(PP_j^{RR}\right)$ must hold. For principal i it is necessary that:

$$E\left(PP_i^{AA}\right) - E\left(PP_i^{RR}\right) = \frac{R\sigma^2\left(2b_jc_i\left(b_i^2+2R\sigma^2\right) - b_i^2\left(b_j^2+2R\sigma^2\right)\right)}{2\left(b_j^2+2R\sigma^2\right)\left(b_i^2+2R\sigma^2\right)} > 0$$

This condition will hold if the numerator is positive. The numerator is positive when:

$$c_i > \frac{b_i^2\left(b_j^2+2R\sigma^2\right)}{2b_j\left(b_i^2+2R\sigma^2\right)} \tag{1.20}$$

Principal j

Principal i	APE	RPE
APE	$F+\frac{b_i^2 b_j^2(b_i^2-2b_ic_j)+2R\sigma^2\left(b_i^4-2b_j^3c_i\right)}{4\left(b_j^2+2R\sigma^2\right)\left(b_i^2+2R\sigma^2\right)}-\hat{u}$, $F+\frac{b_j^2 b_i^2\left(b_j^2-2b_jc_i\right)+2R\sigma^2\left(b_j^4-2b_i^3c_j\right)}{4\left(b_i^2+2R\sigma^2\right)\left(b_j^2+2R\sigma^2\right)}-\hat{u}$	$F-\frac{b_i^2\left(2b_ic_j-b_i^2\right)+4R\sigma^2 b_jc_i}{4\left(b_i^2+2R\sigma^2\right)}-\hat{u}$, $F+\frac{b_i^2\left(b_j^2-2b_ic_j\right)+2R\sigma^2 b_j^2}{4\left(b_i^2+2R\sigma^2\right)}-\hat{u}$
RPE	$F+\frac{b_j^2\left(b_i^2-2b_jc_i\right)+2R\sigma^2 b_i^2}{4\left(b_j^2+2R\sigma^2\right)}-\hat{u}$, $F-\frac{b_j^2\left(2b_jc_i-b_j^2\right)+4R\sigma^2 b_ic_j}{4\left(b_j^2+2R\sigma^2\right)}-\hat{u}$	$F+\frac{1}{4}\left(b_i^2-2b_jc_i\right)$, $F+\frac{1}{4}\left(b_j^2-2b_ic_j\right)$

Figure 1.2: The principals' expected pay-offs in stage 1 when there are negative effort externalities, profit functions are asymmetric and rho=1.

The equivalent condition for $E\left(PP_j^{AA}\right) - E\left(PP_j^{RR}\right) > 0$ is:

$$c_j > \frac{b_j^2\left(b_i^2 + 2R\sigma^2\right)}{2b_i\left(b_j^2 + 2R\sigma^2\right)} \tag{1.21}$$

To demonstrate that (1.20) and (1.21) can hold simultaneously let $b_j = \gamma b_i$. Using this relationship and recalling that $b_i > c_i$ and $b_j > c_j$, for a Pareto improvement to be feasible requires:

$$\frac{b_i\left(2R\sigma^2 + \gamma^2 b_i^2\right)}{2\gamma\left(2R\sigma^2 + b_i^2\right)} < c_i < b_i \tag{1.22}$$

and

$$\frac{b_j\left(b_j^2 + 2R\sigma^2\gamma^2\right)}{2\gamma\left(b_j^2 + 2R\sigma^2\right)} < c_j < b_j \tag{1.23}$$

to both hold. (1.22) is guaranteed to be a non-empty set if:

$$2\left(2\gamma - 1\right)R\sigma^2 + \gamma\left(2 - \gamma\right)b_i^2 > 0 \tag{1.24}$$

and (1.23) is guaranteed to be non-empty if:

$$\left(2\gamma - 1\right)b_j^2 + 2\gamma\left(2 - \gamma\right)R\sigma^2 > 0 \tag{1.25}$$

(1.24) and (1.25) are guaranteed to hold simultaneously if $\gamma \in \left[\frac{1}{2}, 2\right]$. Thus, for $\gamma \in \left[\frac{1}{2}, 2\right]$ it is feasible for (APE, APE) to offer a Pareto improvement over (RPE, RPE).

■

Compared to the condition for a prisoners' dilemma in Proposition 1, both of these conditions are linked to $R\sigma^2$. The derivatives of the lower bounds in (1.22) and (1.23) with respect to $R\sigma^2$ are:

$$\frac{d}{dR\sigma^2}\left(\frac{b_i^2\left(b_j^2+2R\sigma^2\right)}{2b_j\left(b_i^2+2R\sigma^2\right)}\right)=\frac{b_i^2\left(b_i^2-b_j^2\right)}{b_j\left(b_i^2+2R\sigma^2\right)^2}$$

$$\frac{d}{dR\sigma^2}\left(\frac{b_j^2\left(b_i^2+2R\sigma^2\right)}{2b_i\left(b_j^2+2R\sigma^2\right)}\right)=-\frac{b_j^2\left(b_i^2-b_j^2\right)}{b_i\left(b_j^2+2R\sigma^2\right)^2}$$

These derivatives show that the lower bounds for c_i and c_j move in opposite directions when $R\sigma^2$ increases. When $b_j > b_i$, as $R\sigma^2$ increases, the lower bound on c_i relaxes whilst the lower bound on c_j tightens.[53]

1.8.4. Appendix 1.4 - Section 1.5 Proofs

Proof of Lemma 1.1

Assume the principals act as a monopolist to maximise their joint pay-off. Consider incentive contracts allowing RPE.

Each agent's problem is the same as that shown in Section 1.4.1. Hence, when expressed in terms of the optimal contract weights, the optimal effort level is still (1.7) and firm i's expected profits are still (1.8). Summing the expressions for $E\left(PP_i\right)$ and $E\left(PP_j\right)$ gives the joint pay-off maximisation problem as:

$$\max_{\alpha_i,\beta_i,\alpha_j,\beta_j} E\left(PP_i\right)+E\left(PP_j\right)=2F+\frac{1}{2}\left(b^2\alpha_i+c^2\beta_j-bc\alpha_j-bc\beta_i\right)$$

[53]When $b_i > b_j$, as $R\sigma^2$ increases, the lower bound on c_j relaxes whilst the lower bound on c_i tightens.

$$+\frac{1}{2}\left(b^2\alpha_j + c^2\beta_i - bc\alpha_i - bc\beta_j\right) - 2\widehat{u}$$
$$-\left(\frac{1}{2}\left(\alpha_i b - \beta_i c\right)\right)^2 - \left(\frac{1}{2}\left(\alpha_j b - \beta_j c\right)\right)^2$$
$$-\frac{R}{2}\sigma^2\left(\alpha_i^2 + 2\rho\alpha_i\beta_i + \beta_i^2\right) - \frac{R}{2}\sigma^2\left(\alpha_j^2 + 2\rho\alpha_j\beta_j + \beta_j^2\right)$$

Since the firms are symmetric, the optimal solution must involve $\alpha_i = \alpha_j = \alpha$ and $\beta_i = \beta_j = \beta$.[54] Hence, the problem reduces to:

$$\max_{\alpha,\beta} E\left(PP^{ColCol}\right) = 2\begin{pmatrix} F + \frac{1}{2}\left(b^2\alpha + c^2\beta - bc\alpha - bc\beta\right) \\ -\widehat{u} - \left(\frac{1}{2}\left(\alpha b - \beta c\right)\right)^2 \\ -\frac{R}{2}\sigma^2\left(\alpha^2 + 2\rho\alpha\beta + \beta^2\right) \end{pmatrix} \tag{1.26}$$

This problem is concave since:

$$\text{(i)} \quad \frac{\partial^2 E\left(PP^{ColCol}\right)}{\partial\alpha^2} = -\left(b^2 + 2R\sigma^2\right) < 0$$

and

$$\text{(ii)} \quad \left(\frac{\partial^2 E\left(PP^{ColCol}\right)}{\partial\alpha^2}\right)\left(\frac{\partial^2 E\left(PP^{ColCol}\right)}{\partial\beta^2}\right) - \left(\frac{\partial^2 E\left(PP^{ColCol}\right)}{\partial\alpha\partial\beta}\right)^2 =$$
$$2R\sigma^2\left(2R\sigma^2\left(1-\rho^2\right) + b^2 + c^2 + 2bc\rho\right) > 0$$

The FOCs for this problem are:

$$\frac{\partial E\left(PP^{ColCol}\right)}{\partial\alpha} = b^2 - bc - b^2\alpha + bc\beta - 2R\alpha\sigma^2 - 2R\sigma^2\beta\rho = 0$$

$$\frac{\partial E\left(PP^{ColCol}\right)}{\partial\beta} = c^2 - bc - c^2\beta + bc\alpha - 2R\sigma^2\beta - 2R\alpha\sigma^2\rho = 0$$

Solving these FOCs as a pair of simultaneous equations gives:

[54] As the cost of effort is convex and the risk premium paid to agents depends on squared terms of the contract weights, setting asymmetric contract weights could never minimise costs.

$$\alpha^{Col} = \frac{(b-c)(b+c\rho)}{2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho}$$

$$\beta^{Col} = -\frac{(b-c)(c+b\rho)}{2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho}$$

Inserting α^{Col} and β^{Col} back into $e_i = \frac{1}{2}(\alpha_i b - \beta_i c)$ gives each agent's optimal effort level as:

$$e^{Col} = \frac{(b-c)(b^2+c^2+2bc\rho)}{2(2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho)}$$

■

Proof of Lemma 1.2

Inserting α^{Col} and β^{Col} into the objective function of (1.26) and dividing by two gives each principal's expected pay-off as:

$$E\left(PP^{ColCol}\right) = F + \frac{(b-c)^2(b^2+c^2+2bc\rho)}{4(2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho)} - \widehat{u}$$

$E\left(PP^{ColCol}\right) - E\left(PP^{AA}\right)$ is given by:

$$E\left(PP^{ColCol}\right) - E\left(PP^{AA}\right) = \frac{(b-c)^2(b^2+c^2+2bc\rho)}{4(2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho)} - \frac{b^3(b-2c)}{4(2R\sigma^2+b^2)}$$

The first term on the RHS is always positive. The second term on the RHS can be positive or negative. When $b < 2c$, it is positive and, when $b = 2c$, the term is zero. Hence, when $b \leq 2c$, $E\left(PP^{ColCol}\right) > E\left(PP^{AA}\right) \geq E\left(PP^{RR}\right)$.

Now consider when $b > 2c$. From the proof of Proposition 1.1 when $b > 2c$ $E\left(PP^{RR}\right) > E\left(PP^{AA}\right)$. So if $E\left(PP^{ColCol}\right) > E\left(PP^{RR}\right)$ it implies $E\left(PP^{ColCol}\right) > E\left(PP^{AA}\right)$. The expression for $E\left(PP^{ColCol}\right) - E\left(PP^{RR}\right)$ is:

$$E\left(PP^{ColCol}\right) - E\left(PP^{RR}\right) = \frac{c^2\left(b^2 + c^2 + 2bc\rho\right)}{4\left(2R\sigma^2(1-\rho^2) + b^2 + c^2 + 2bc\rho\right)}$$

This expression is always positive and so $E\left(PP^{ColCol}\right) > E\left(PP^{RR}\right) > E\left(PP^{AA}\right)$. ■

Proof of Proposition 1.2

From Section 1.5 a "Grim-Trigger" strategy can support $(Collude, Collude)$ as a subgame perfect Nash equilibrium if δ satisfies:

$$\sum_{t=0}^{\infty} \delta^t E\left(PP^{ColCol}\right) > E\left(PP^{RCol}\right) + \sum_{t=1}^{\infty} \delta^t E\left(PP^{RR}\right)$$

Noting that:

$$\sum_{t=0}^{\infty} \delta^t = 1 + \delta + \delta^2 + \delta^3 = \frac{1}{1-\delta}$$

the inequality above can be re-written as:

$$\frac{1}{1-\delta} E\left(PP^{ColCol}\right) > E\left(PP^{RCol}\right) + \frac{\delta}{1-\delta} E\left(PP^{RR}\right)$$

Substituting in the expressions for $E\left(PP^{ColCol}\right)$, $E\left(PP^{RCol}\right)$[55] and $E\left(PP^{RR}\right)$ and cancelling $\frac{1}{1-\delta}\left(F - \widehat{u}\right)$ gives:

[55]The expected pay-off $E\left(PP^{RCol}\right)$ can be found by inserting $\alpha_i = \alpha^R$, $\beta_i = \beta^R$, $\alpha_j = \alpha^{Col}$ and $\beta_j = \beta^{Col}$ into the objective function of (1.10).

$$\frac{1}{1-\delta}\left(\frac{1}{4}(b-c)^2\frac{b^2+c^2+2bc\rho}{2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho}\right) >$$

$$\frac{(b^2+2c^2-2bc)(b^2+c^2+2bc\rho)}{4(2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho)}+\frac{\delta}{1-\delta}\left(\frac{b(b-2c)(b^2+2\rho bc+c^2)}{4(2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho)}\right)$$

Re-arranging to make δ the subject gives:

$$\delta > \frac{1}{2}$$

■

Partial Derivatives of $E\left(PP^{ColCol}\right) - E\left(PP^{RR}\right)$

The expression for the per period gain from contract weight collusion is:

$$E\left(PP^{ColCol}\right) - E\left(PP^{RR}\right) = \frac{c^2(b^2+c^2+2bc\rho)}{4(2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho)}$$

The partial derivatives of $E\left(PP^{ColCol}\right) - E\left(PP^{RR}\right)$ are:

$$\frac{\partial\left(E\left(PP^{ColCol}\right) - E\left(PP^{RR}\right)\right)}{\partial c} =$$

$$\frac{c\left((b^2+c^2+2bc\rho)^2+2R\sigma^2(1-\rho^2)(b^2+2c^2+3bc\rho)\right)}{2(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} > 0$$

$$\frac{\partial\left(E\left(PP^{ColCol}\right) - E\left(PP^{RR}\right)\right)}{\partial \rho} = \frac{c^2R\sigma^2(b^2\rho+bc\rho^2+bc+c^2\rho)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} > 0$$

$$\frac{\partial\left(E\left(PP^{ColCol}\right) - E\left(PP^{RR}\right)\right)}{\partial R\sigma^2} = \frac{8c^2(1-\rho^2)(b^2+2\rho bc+c^2)}{16(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} < 0$$

Proof of Proposition 1.3

From Section 1.5, and Friedman (1971), collusion to not use RPE is sustainable if:

$$\sum_{t=0}^{\infty} \delta^t E\left(PP^{AA}\right) > E\left(PP^{RA}\right) + \sum_{t=1}^{\infty} \delta^t E\left(PP^{RR}\right)$$

Substituting in the expressions for the expected pay-offs and cancelling $\frac{1}{1-\delta}\left(F - \widehat{u}\right)$ gives:

$$\frac{1}{1-\delta}\left(\frac{b^3\left(b-2c\right)}{4\left(2R\sigma^2+b^2\right)}\right) >$$

$$\frac{b^2\left(2R\sigma^2\left(b^2+2bc\rho^2+2bc\rho-2bc+c^2\right)+b\left(b-2c\right)\left(b^2+c^2+2bc\rho\right)\right)}{4\left(b^2+2R\sigma^2\right)\left(b^2+2bc\rho+c^2-2R\sigma^2\rho^2+2R\sigma^2\right)}$$

$$+\frac{\delta}{1-\delta}\left(\frac{b\left(b-2c\right)\left(b^2+2\rho bc+c^2\right)}{4\left(2R\sigma^2(1-\rho^2)+b^2+c^2+2bc\rho\right)}\right)$$

Simplifying gives the critical discount factor as:

$$\delta > \frac{b}{2c}$$

■

1.8.5. Appendix 1.5 - Positive Effort Externalities

Solution Following (RPE, RPE)

Following (RPE, RPE) agent i's stage 3 maximisation problem is:

$$\max_{e_i} CE_i = w_i + \alpha_i\left(F + be_i + de_j\right) + \beta_i\left(F + be_j + de_i\right) \tag{1.27}$$

$$-e_i^2 - \frac{R}{2}\sigma^2\left(\alpha_i^2 + 2\rho\alpha_i\beta_i + \beta_i^2\right)$$

This problem is concave in e_i. Re-arranging the resulting FOC gives agent i's optimal effort as:

$$e_i = \frac{1}{2}\left(b\alpha_i + d\beta_i\right) \tag{1.28}$$

Principal i's stage 2 problem is to maximise $E\left(PP_i\right)$ subject to the ICC and PC. This constrained maximisation problem can be reduced to the following unconstrained maximisation problem:

$$\max_{\alpha_i,\beta_i} E\left(PP_i\right) = F + \frac{1}{2}\left(b^2\alpha_i + d^2\beta_j + bd\alpha_j + bd\beta_i\right) \tag{1.29}$$

$$-\widehat{u} - \left(\frac{1}{2}\left(\alpha_i b + \beta_i d\right)\right)^2 - \frac{R}{2}\sigma^2\left(\alpha_i^2 + 2\rho\alpha_i\beta_i + \beta_i^2\right)$$

Concavity requires that:

$$\text{(i)}\ \frac{\partial^2 E(PP_i)}{\partial\alpha_i^2} = -\left(\tfrac{1}{2}b^2 + R\sigma^2\right) < 0$$

$$\text{(ii)}\left(\frac{\partial^2 E(PP_i)}{\partial\alpha_i^2}\right)\left(\frac{\partial^2 E(PP_i)}{\partial\beta_i^2}\right) - \left(\frac{\partial^2 E(PP_i)}{\partial\alpha_i\partial\beta_i}\right)^2$$

$$= \tfrac{1}{2}R\sigma^2\left(2R\sigma^2\left(1-\rho^2\right) + b^2 + d^2 - 2bd\rho\right) > 0$$

Both of these conditions always hold. Condition (ii) holds due to the starting assumptions that $\rho \in [0,1]$ and $b > d$. $b > d$ implies $\left(b-d\right)^2 = b^2 - 2bd + d^2 > 0$ and $\rho \in [0,1]$ implies $b^2 + d^2 - 2bd\rho \geq \left(b-d\right)^2 > 0$. Hence (1.29) is concave.

The FOCs for this maximisation problem are:

$$\frac{\partial E\left(PP_i\right)}{\partial\alpha_i} = \frac{1}{2}b^2 - \frac{1}{2}b^2\alpha_i - \frac{1}{2}bd\beta_i - R\sigma^2\alpha_i - R\sigma^2\rho\beta_i = 0$$

$$\frac{\partial E\left(PP_i\right)}{\partial\beta_i} = \frac{1}{2}bd - \frac{1}{2}d^2\beta_i - \frac{1}{2}bd\alpha_i - R\sigma^2\beta_i - R\sigma^2\rho\alpha_i = 0$$

Solving these FOCs as a pair of simultaneous equations gives:

$$\alpha_{+}^{R} = \frac{b\left(b - d\rho\right)}{2R\sigma^{2}\left(1-\rho^{2}\right) + b^{2} + d^{2} - 2bd\rho}$$

$$\beta_{+}^{R} = \frac{b\left(d - b\rho\right)}{2R\sigma^{2}\left(1-\rho^{2}\right) + b^{2} + d^{2} - 2bd\rho}$$

Inserting α_{+}^{R} and β_{+}^{R} into (1.28) gives agent i's optimal effort as:

$$e_{+}^{R} = \frac{b\left(b^{2} + d^{2} - 2bd\rho\right)}{2\left(2R\sigma^{2}\left(1-\rho^{2}\right) + b^{2} + d^{2} - 2bd\rho\right)}$$

By symmetry, the same values also hold for principal-agent pair j. Inserting α_{+}^{R}, β_{+}^{R} and e_{+}^{R} back into the objective function of (1.27) and noting $CE_{i} = \widehat{u}$, gives the optimal base wage as:

$$w_{+}^{RR} = \widehat{u} + \left(\frac{b\left(1-\rho\right)\left(b+d\right)}{2R\sigma^{2}\left(1-\rho^{2}\right) + b^{2} + d^{2} - 2bd\rho}\right)F$$

$$+\frac{b^{2}\left(\left(b^{2}+d^{2}\right)\left(2\rho-1\right) - 2bd\left(2-\rho\right)\right)\left(b^{2}+d^{2}-2bd\rho\right)}{4\left(2R\sigma^{2}\left(1-\rho^{2}\right) + b^{2} + d^{2} - 2bd\rho\right)^{2}}$$

$$+\frac{R\sigma^{2}b^{2}\left(1-\rho^{2}\right)\left(b^{2}+d^{2}-2bd\rho\right)}{2\left(2R\sigma^{2}\left(1-\rho^{2}\right) + b^{2} + d^{2} - 2bd\rho\right)^{2}}$$

Inserting α_{+}^{R} and β_{+}^{R} into the objective function of (1.29) gives principal i's expected pay-off as:

$$E\left(PP_{+}^{RR}\right) = F + \frac{b\left(b+2d\right)\left(b^{2}+d^{2}-2bd\rho\right)}{4\left(2R\sigma^{2}\left(1-\rho^{2}\right) + b^{2} + d^{2} - 2bd\rho\right)} - \widehat{u}$$

Solution Following (APE, APE)

Following (APE, APE) agent i's stage 3 maximisation problem is:

$$\max_{e_i} CE_i = w_i + \alpha_i \left(F + be_i + de_j\right) - e_i^2 - \frac{R}{2}\alpha_i^2\sigma^2 \tag{1.30}$$

This problem is concave in e_i. Re-arranging the resulting FOC gives agent i's optimal effort as:

$$e_i = \frac{1}{2}b\alpha_i \tag{1.31}$$

Principal i's unconstrained maximisation problem is:

$$\max_{\alpha_i} E(PP_i) = F + \frac{1}{2}b\left(b\alpha_i + d\alpha_j\right) - \widehat{u} - \left(\frac{1}{2}b\alpha_i\right)^2 - \frac{R}{2}\alpha_i^2\sigma^2 \tag{1.32}$$

This problem is concave in α_i. Re-arranging the FOC gives the optimal value of α_i as:

$$\alpha_+^A = \frac{b^2}{2R\sigma^2 + b^2}$$

Inserting α_+^A into (1.31) the agent's optimal effort is:

$$e_+^A = \frac{1}{2}b\left(\frac{b^2}{2R\sigma^2 + b^2}\right)$$

Inserting α_+^A and e_+^A back into the objective function of (1.30) and setting equal to $\widehat{u}$ gives:

$$w_+^{AA} = \widehat{u} - \frac{b^2}{2R\sigma^2 + b^2}\left(F + \frac{b^3\left(b + 2d\right)}{4\left(2R\sigma^2 + b^2\right)} - \frac{b^2 R\sigma^2}{2\left(2R\sigma^2 + b^2\right)}\right)$$

Inserting α_+^A back into the objective function of (1.32) gives principal i's expected pay-off as:

$$E(PP_{+}^{AA}) = F + \frac{1}{4} b^{3} \frac{b+2d}{2R\sigma^{2}+b^{2}} - \widehat{u}$$

Solutions Following (RPE, APE) **and** (APE, RPE)

Following (RPE, APE) principal i's expected pay-off, expressed in terms of the contract weights, is given by (1.29) with the restriction $\beta_j = 0$ imposed. Noting that $\alpha_i = \alpha_{+}^{R}$, $\beta_i = \beta_{+}^{R}$ and $\alpha_j = \alpha_{+}^{A}$ principal i's expected pay-off is:

$$E\left(PP_{+}^{RA}\right) = F - \widehat{u} +$$

$$\frac{b^{2}\left(\left(b\left(b+2d\right)+2R\sigma^{2}\right)\left(b^{2}+d^{2}\right)-2bd\left(b\rho\left(b+2d\right)+2R\sigma^{2}\left(\rho^{2}+\rho-1\right)\right)\right)}{4\left(2R\sigma^{2}+b^{2}\right)\left(2R\sigma^{2}\left(1-\rho^{2}\right)+b^{2}+d^{2}-2bd\rho\right)}$$

Inserting $\alpha_i = \alpha_{+}^{R}$, $\beta_i = \beta_{+}^{R}$, $e_i = e_{+}^{R}$ and $e_j = e_{+}^{A}$ back into the objective function of (1.27) and setting equal to $\widehat{u}$, gives the optimal base wage as:

$$w_{i+}^{RA} = \widehat{u} + \frac{b\left(1-\rho\right)\left(b+d\right)}{2R\sigma^{2}\left(1-\rho^{2}\right)+b^{2}+d^{2}-2bd\rho} F$$

$$+\left(\frac{b^{2}}{2R\sigma^{2}\left(1-\rho^{2}\right)+b^{2}+d^{2}-2bd\rho}\right)$$

$$\left(\frac{b^{2}\left(b^{2}\rho+d^{2}\rho-2bd\right)}{2\left(2R\sigma^{2}+b^{2}\right)} + \frac{3\left(b^{2}+d^{2}-2bd\rho\right)^{2}}{4\left(2R\sigma^{2}\left(1-\rho^{2}\right)+b^{2}+d^{2}-2bd\rho\right)}\right)$$

$$+\frac{R\sigma^{2}b^{2}\left(1-\rho^{2}\right)\left(b^{2}+d^{2}-2bd\rho\right)}{2\left(2R\sigma^{2}\left(1-\rho^{2}\right)+b^{2}+d^{2}-2bd\rho\right)^{2}}$$

Following (APE, RPE), principal i's expected pay-off is given by (1.29) with the restriction $\beta_i = 0$ imposed. Noting that $\alpha_i = \alpha_{+}^{A}$, $\alpha_j = \alpha_{+}^{R}$ and $\beta_i = \beta_{+}^{R}$, principal i's expected pay-off becomes:

$$E\left(PP_{+}^{AR}\right) = F - \widehat{u} +$$

$$\frac{b\left(b^2\left(b+2d\right)\left(b^2+d^2-2bd\rho\right)-2R\sigma^2\left(b^3\rho^2-2b^2d-b^3-2d^3+4bd^2\rho\right)\right)}{4\left(2R\sigma^2+b^2\right)\left(2R\sigma^2\left(1-\rho^2\right)+b^2+d^2-2bd\rho\right)}$$

Using all of the principals' expected pay-offs, the pay-off matrix in Figure 1.3 can be formed. Inserting $\alpha_i = \alpha_+^A$, $e_i = e_+^A$ and $e_j = e_+^R$ back into (1.30) and setting equal to $\widehat{u}$, gives the optimal base wage as:

$$w_{i+}^{AR} = \widehat{u} - \frac{db\left(b^2+d^2-2bd\rho\right)}{2\left(2R\sigma^2\left(1-\rho^2\right)+b^2+d^2-2bd\rho\right)}$$

$$+\left(\frac{b^2}{2R\sigma^2+b^2}\right)\left(\frac{b^4+2b^2R\sigma^2-\left(4F+2b^2\right)\left(2R\sigma^2+b^2\right)}{4\left(2R\sigma^2+b^2\right)}\right)$$

Principal i \ Principal j	APE	RPE
APE	$F+\frac{b^3(b+2d)}{4(2R\sigma^2+b^2)}-\hat{u}$, $F+\frac{b^3(b+2d)}{4(2R\sigma^2+b^2)}-\hat{u}$	$F+\frac{b(b^2(b+2d)(b^2+d^2-2bd\rho)-2R\sigma^2(b^3\rho^2-2b^2d-b^3-2d^3+4bd^2\rho}{4(2R\sigma^2+b^2)(2R\sigma^2(1-\rho^2)+b^2+d^2-2bd\rho)}-\hat{u}$, $F+\frac{b^2((b(b+2d)+2R\sigma^2)(b^2+d^2)-2bd(b\rho(b+2d)+2R\sigma^2(\rho^2+\rho-1)))}{4(2R\sigma^2+b^2)(2R\sigma^2(1-\rho^2)+b^2+d^2-2bd\rho)}-\hat{u}$
RPE	$F+\frac{b^2((b(b+2d)+2R\sigma^2)(b^2+d^2)-2bd(b\rho(b+2d)+2R\sigma^2(\rho^2+\rho-1)))}{4(2R\sigma^2+b^2)(2R\sigma^2(1-\rho^2)+b^2+d^2-2bd\rho)}-\hat{u}$, $F+\frac{b(b^2(b+2d)(b^2+d^2-2bd\rho)-2R\sigma^2(b^3\rho^2-2b^2d-b^3-2d^3+4bd^2\rho}{4(2R\sigma^2+b^2)(2R\sigma^2(1-\rho^2)+b^2+d^2-2bd\rho)}-\hat{u}$	$F+\frac{b(b+2d)(b^2+d^2-2bd\rho)}{4(2R\sigma^2(1-\rho^2)+b^2+d^2-2bd\rho)}-\hat{u}$, $F+\frac{b(b+2d)(b^2+d^2-2bd\rho)}{4(2R\sigma^2(1-\rho^2)+b^2+d^2-2bd\rho)}-\hat{u}$

Figure 1.3: The principals' expected pay-offs in stage 1 when there are positive effort externalities.

1.8.6. Appendix 1.6 - Comparative Statics

Summary - Negative Effort Externalities

e **- APE**

$$\frac{\partial e^A}{\partial b} = \frac{b^2\left(b^2+6R\sigma^2\right)}{2(b^2+2R\sigma^2)^2} > 0 \qquad \frac{\partial e^A}{\partial R\sigma^2} = -\frac{b^3}{(b^2+2R\sigma^2)^2} < 0$$

e **- RPE**

$$\frac{\partial e^R}{\partial b} = \frac{\left(b^2+c^2+2bc\rho\right)^2+2R\sigma^2(1-\rho^2)\left(3b^2+4bc+c^2\right)}{2(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} > 0 \qquad \frac{\partial e^R}{\partial c} = \frac{2bR\sigma^2(c+b\rho)\left(1-\rho^2\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} > 0$$

$$\frac{\partial e^R}{\partial \rho} = \frac{2bR\sigma^2\left(\left(b^2+c^2\right)\rho+bc\left(1+\rho^2\right)\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} > 0 \qquad \frac{\partial e^R}{\partial R\sigma^2} = -\frac{b(1-\rho^2)\left(b^2+2\rho bc+c^2\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} < 0$$

e **- Contract Weight Collusion**

$$\frac{\partial e^{Col}}{\partial b} = \frac{\left(b^2+c^2+2bc\rho\right)^2+2R\sigma^2\left(1-\rho^2\right)\left(3b^2-2bc+c^2+2c\rho(2b-c)\right)}{2(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} > 0$$

$$\frac{\partial e^{Col}}{\partial c} = -\frac{\left(b^2+c^2+2bc\rho\right)^2-2R\sigma^2(1-\rho^2)\left(b(2\rho-1)(b-2c)-3c^2\right)}{2(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} \gtrless 0$$

$$\frac{\partial e^{Col}}{\partial \rho} = \frac{2R\sigma^2(b-c)\left(b^2\rho+bc\rho^2+bc+c^2\rho\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} > 0 \qquad \frac{\partial e^{Col}}{\partial R\sigma^2} = -\frac{(b-c)(1-\rho^2)\left(b^2+2\rho bc+c^2\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} < 0$$

α **- APE**

$$\frac{\partial \alpha^A}{\partial b} = \frac{4bR\sigma^2}{(b^2+2R\sigma^2)^2} > 0 \qquad \frac{\partial \alpha^A}{\partial R\sigma^2} = -\frac{2b^2}{(b^2+2R\sigma^2)^2} < 0$$

α **- RPE**

$$\frac{\partial \alpha^R}{\partial b} = \frac{b^2c\rho+2bc^2+4bR\sigma^2\left(1-\rho^2\right)+c^3\rho+2cR\sigma^2\rho\left(1-\rho^2\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} > 0$$

$$\frac{\partial \alpha^R}{\partial c} = -\frac{b\left(b^2\rho+2bc+c^2\rho-2R\sigma^2\rho\left(1-\rho^2\right)\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} \gtrless 0$$

$$\frac{\partial \alpha^R}{\partial \rho} = \frac{b\left(c\left(c^2-b^2\right)+2R\sigma^2\left(c+2b\rho+c\rho^2\right)\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} \gtrless 0 \qquad \frac{\partial \alpha^R}{\partial R\sigma^2} = -\frac{2(b+c\rho)\left(1-\rho^2\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} < 0$$

α - Contract Weight Collusion

$$\frac{\partial \alpha^{Col}}{\partial b} = \frac{(1+\rho)\left(c\left(2c^2\rho+2bc+b^2-c^2\right)+2R\sigma^2(1-\rho)(2b-c+c\rho)\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} > 0$$

$$\frac{\partial \alpha^{Col}}{\partial c} = -\frac{(1+\rho)\left(b\left(2c^2\rho+2bc+b^2-c^2\right)+2R\sigma^2(1-\rho)((1-\rho)b+2c\rho)\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} < 0$$

$$\frac{\partial \alpha^{Col}}{\partial \rho} = \frac{(b-c)\left(2R\sigma^2\left(c+2b\rho+c\rho^2\right)-c\left(b^2-c^2\right)\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} \gtrless 0 \qquad \frac{\partial \alpha^{Col}}{\partial R\sigma^2} = -\frac{2\left(1-\rho^2\right)(b-c)(b+c\rho)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} < 0$$

β - RPE

$$\frac{\partial \beta^{R}}{\partial b} = -\frac{\left(c\left(c^2+2bc\rho-b^2\left(1-2\rho^2\right)\right)+4Rb\sigma^2\rho(1-\rho^2)+2Rc\sigma^2(1-\rho^2)\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} \gtrless 0$$

$$\frac{\partial \beta^{R}}{\partial c} = \frac{b\left(2bc\rho+c^2-b^2\left(1-2\rho^2\right)-2R\sigma^2(1-\rho^2)\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} \gtrless 0$$

$$\frac{\partial \beta^{R}}{\partial \rho} = -\frac{b\left(b^3-bc^2+2R\sigma^2\left(b+2c\rho+b\rho^2\right)\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} < 0 \qquad \frac{\partial \beta^{R}}{\partial R\sigma^2} = \frac{2b(1-\rho^2)(c+b\rho)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} > 0$$

β - Contract Weight Collusion

$$\frac{\partial \beta^{Col}}{\partial b} = -\frac{(1+\rho)\left(c\left(2b^2\rho+2bc-b^2+c^2\right)+2R\sigma^2(1-\rho)(c+2b\rho-c\rho)\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} \gtrless 0$$

$$\frac{\partial \beta^{Col}}{\partial c} = \frac{(1+\rho)\left(b\left(2b^2\rho+2bc-b^2+c^2\right)+2R\sigma^2(1-\rho)(2c-b+b\rho)\right)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} \gtrless 0$$

$$\frac{\partial \beta^{Col}}{\partial \rho} = -\frac{(b-c)\left(b^3-bc^2+2R\sigma^2\left(b+2c\rho+b\rho^2\right)\right)}{(b^2+2bc\rho+c^2-2R\sigma^2\rho^2+2R\sigma^2)^2} < 0 \qquad \frac{\partial \beta^{Col}}{\partial R\sigma^2} = \frac{2\left(1-\rho^2\right)(b-c)(c+b\rho)}{(b^2+2bc\rho+c^2+2R\sigma^2(1-\rho^2))^2} > 0$$

Summary - Positive Effort Externalities

e - RPE

$$\frac{\partial e_+^R}{\partial b} = \frac{\left(b^2+d^2-2bd\rho\right)^2+2R\sigma^2(1-\rho^2)\left(3b^2+d^2-4bd\rho\right)}{2(b^2-2bd\rho+d^2+2R\sigma^2(1-\rho^2))^2} > 0$$

$$\frac{\partial e_+^R}{\partial d} = \frac{2bR\sigma^2(d-b\rho)\left(1-\rho^2\right)}{(b^2-2bd\rho+d^2+2R\sigma^2(1-\rho^2))^2} \gtrless 0$$

$$\frac{\partial e_+^R}{\partial \rho} = \frac{2bR\sigma^2(b-d\rho)(b\rho-d)}{(b^2-2bd\rho+d^2+2R\sigma^2(1-\rho^2))^2} \gtrless 0 \qquad \frac{\partial e_+^R}{\partial R\sigma^2} = -\frac{b(1-\rho^2)\left(b^2-2\rho bd+d^2\right)}{(b^2-2bd\rho+d^2+2R\sigma^2(1-\rho^2))^2} < 0$$

α - RPE

$$\frac{\partial \alpha_+^R}{\partial b} = -\frac{d\left(b^2\rho+d^2\rho-2bd\right)-2R\sigma^2(1-\rho^2)(2b-d\rho)}{(b^2-2bd\rho+d^2+2R\sigma^2(1-\rho^2))^2} \gtrless 0 \qquad \frac{\partial \alpha_+^R}{\partial d} = b\frac{b^2\rho+d^2\rho-2bd-2R\sigma^2\rho(1-\rho^2)}{(2R\sigma^2(1-\rho^2)+b^2+d^2-2bd\rho)^2} \gtrless 0$$

$$\frac{\partial \alpha_+^R}{\partial \rho} = -\frac{b\left(d^3-b^2d-4bR\sigma^2\rho+2dR\sigma^2(1-\rho^2)\right)}{(b^2-2bd\rho+d^2+2R\sigma^2(1-\rho^2))^2} \gtrless 0 \qquad \frac{\partial \alpha_+^R}{\partial R\sigma^2} = -\frac{2b(1-\rho^2)(b-d\rho)}{(b^2-2bd\rho+d^2+2R\sigma^2(1-\rho^2))^2} < 0$$

β - RPE

$$\frac{\partial\beta_+^R}{\partial b} = -\frac{d\left(b^2\left(1-2\rho^2\right)+d(2b\rho-d)\right)+2R\sigma^2(1-\rho^2)(2b\rho-d)}{(b^2-2bd\rho+d^2+2R\sigma^2(1-\rho^2))^2} \gtrless 0$$

$$\frac{\partial\beta_+^R}{\partial d} = \frac{b\left(b^2-d^2-2b\rho(b\rho-d)+2R\sigma^2(1-\rho^2)\right)}{(b^2-2bd\rho+d^2+2R\sigma^2(1-\rho^2))^2} \gtrless 0$$

$$\frac{\partial\beta_+^R}{\partial \rho} = -\frac{b\left(b^3-bd^2+2R\sigma^2\left(b-2d\rho+b\rho^2\right)\right)}{(b^2-2bd\rho+d^2+2R\sigma^2(1-\rho^2))^2} < 0 \qquad \frac{\partial\beta_+^R}{\partial R\sigma^2} = -\frac{2b(1-\rho^2)(d-b\rho)}{(b^2-2bd\rho+d^2+2R\sigma^2(1-\rho^2))^2} \gtrless 0$$

Summary when $\rho = 0$

For brevity, when $\rho = 0$ the derivatives with respect to $R\sigma^2$ are not reported. The signs of the derivatives with respect to $R\sigma^2$ do not change from those above.

Negative Effort Externalities When $\rho = 0$, α^R, β^R and e^R simplify to:

$$\alpha^R = \frac{b^2}{2R\sigma^2+b^2+c^2} > 0 \qquad \beta^R = -\frac{bc}{2R\sigma^2+b^2+c^2} < 0 \qquad e^R = \frac{b\left(b^2+c^2\right)}{2(2R\sigma^2+b^2+c^2)}$$

The derivatives of α^R, β^R and e^R with respect to b and c are:

$$\frac{\partial e^R}{\partial b} = \frac{\left(\left(b^2+c^2\right)^2+2R\sigma^2\left(3b^2+c^2\right)\right)}{2(b^2+c^2+2R\sigma^2)^2} > 0 \qquad \frac{\partial e^R}{\partial c} = \frac{2bcR\sigma^2}{(b^2+c^2+2R\sigma^2)^2} > 0$$

$$\frac{\partial\alpha^R}{\partial b} = \frac{2b\left(c^2+2R\sigma^2\right)}{(b^2+c^2+2R\sigma^2)^2} > 0 \qquad \frac{\partial\alpha^R}{\partial c} = -\frac{2b^2c}{(b^2+c^2+2R\sigma^2)^2} < 0$$

$$\frac{\partial\beta^R}{\partial b} = -\frac{c\left(c^2-b^2+2R\sigma^2\right)}{(b^2+c^2+2R\sigma^2)^2} \gtrless 0 \qquad \frac{\partial\beta^R}{\partial c} = -\frac{b\left(b^2-c^2+2R\sigma^2\right)}{(b^2+c^2+2R\sigma^2)^2} < 0$$

When $\rho = 0$, α^{Col}, β^{Col} and e^{Col} simplify to:

$$\alpha^{Col} = \frac{b(b-c)}{2R\sigma^2+b^2+c^2} > 0 \qquad \beta^{Col} = -\frac{c(b-c)}{2R\sigma^2+b^2+c^2} < 0 \qquad e^{Col} = \frac{(b-c)\left(b^2+c^2\right)}{2(2R\sigma^2+b^2+c^2)} > 0$$

The derivatives of α^{Col}, β^{Col} and e^{Col} with respect to b and c are:

$$\frac{\partial e^{Col}}{\partial b} = \frac{\left(b^2+c^2\right)^2+2R\sigma^2\left(3b^2-2bc+c^2\right)}{2(b^2+c^2+2R\sigma^2)^2} > 0 \qquad \frac{\partial e^{Col}}{\partial c} = -\frac{\left(b^2+c^2\right)^2+2R\sigma^2\left(b^2-2bc+3c^2\right)}{2(b^2+c^2+2R\sigma^2)^2} < 0$$

$$\frac{\partial\alpha^{Col}}{\partial b} = \frac{\left(b^2c+2bc^2-c^3+2R\sigma^2(2b-c)\right)}{(b^2+c^2+2R\sigma^2)^2} > 0 \qquad \frac{\partial\alpha^{Col}}{\partial c} = -\frac{b\left(b^2+2bc-c^2+2R\sigma^2\right)}{(b^2+c^2+2R\sigma^2)^2} < 0$$

$$\frac{\partial\beta^{Col}}{\partial b} = -\frac{c\left(2bc-b^2+c^2+2R\sigma^2\right)}{(b^2+c^2+2R\sigma^2)^2} \gtrless 0 \qquad \frac{\partial\beta^{Col}}{\partial c} = \frac{b\left(2bc-b^2+c^2\right)-2R\sigma^2(b-2c)}{(b^2+c^2+2R\sigma^2)^2} \gtrless 0$$

Positive Effort Externalities When $\rho = 0$, α_+^R, β_+^R and e_+^R simplify to:

$$\alpha_+^R = \frac{b^2}{2R\sigma^2+b^2+d^2} > 0 \qquad \beta_+^R = \frac{bd}{2R\sigma^2+b^2+d^2} > 0 \qquad e_+^R = \frac{b\left(b^2+d^2\right)}{2(b^2+d^2+2R\sigma^2)}$$

The derivatives with respect to b and d are:

$$\frac{\partial e_+^R}{\partial b} = \frac{\left(b^2+d^2\right)^2+2R\sigma^2\left(3b^2+d^2\right)}{2(b^2+d^2+2R\sigma^2)^2} > 0 \qquad \frac{\partial e_+^R}{\partial d} = \frac{2R\sigma^2 bd}{(b^2+d^2+2R\sigma^2)^2} > 0$$

$$\frac{\partial \alpha_+^R}{\partial b} = \frac{2b\left(d^2+2R\sigma^2\right)}{(b^2+d^2+2R\sigma^2)^2} > 0 \qquad \frac{\partial \alpha_+^R}{\partial d} = -\frac{2b^2 d}{(b^2+d^2+2R\sigma^2)^2} < 0$$

$$\frac{\partial \beta_+^R}{\partial b} = \frac{d\left(d^2-b^2+2R\sigma^2\right)}{(b^2+d^2+2R\sigma^2)^2} \gtrless 0 \qquad \frac{\partial \beta_+^R}{\partial d} = \frac{b\left(b^2-d^2+2R\sigma^2\right)}{(b^2+d^2+2R\sigma^2)^2} > 0$$

Summary when $\rho = 1$

When $\rho = 1$, RPE means all the optimal contract weights, and their derivatives, are independent of $R\sigma^2$.

Negative Effort Externalities When $\rho = 1$, the expressions for α^R, β^R and e^R simplify to:

$$\alpha^R = \tfrac{b}{b+c} > 0 \qquad \beta^R = -\tfrac{b}{b+c} < 0 \qquad e^R = \tfrac{1}{2}b$$

The derivatives of α^R, β^R and e^R with respect to b and c are:

$$\frac{\partial e^R}{\partial b} = \frac{1}{2} > 0 \qquad \frac{\partial e^R}{\partial c} = 0$$

$$\frac{\partial \alpha^R}{\partial b} = \frac{c}{(b+c)^2} > 0 \qquad \frac{\partial \alpha^R}{\partial c} = -\frac{b}{(b+c)^2} < 0$$

$$\frac{\partial \beta^R}{\partial b} = -\frac{c}{(b+c)^2} < 0 \qquad \frac{\partial \beta^R}{\partial c} = \frac{b}{(b+c)^2} > 0$$

When $\rho = 1$, α^{Col}, β^{Col} and e^{Col} simplify to:

$$\alpha^{Col} = \tfrac{b-c}{b+c} > 0 \qquad \beta^{Col} = -\tfrac{b-c}{b+c} < 0 \qquad e^{Col} = \tfrac{1}{2}\left(b-c\right)$$

The derivatives of α^{Col}, β^{Col} and e^{Col} with respect to b and c are:

$$\frac{\partial e^{Col}}{\partial b} = \frac{1}{2} > 0 \qquad \frac{\partial e^{Col}}{\partial c} = -\frac{1}{2} < 0$$

$$\frac{\partial \alpha^{Col}}{\partial b} = \frac{2c}{(b+c)^2} > 0 \qquad \frac{\partial \alpha^{Col}}{\partial c} = -\frac{2b}{(b+c)^2} < 0$$

$$\frac{\partial \beta^{Col}}{\partial b} = -\frac{2c}{(b+c)^2} < 0 \qquad \frac{\partial \beta^{Col}}{\partial c} = \frac{2b}{(b+c)^2} > 0$$

Positive Effort Externalities When $\rho = 1$, α_+^R, β_+^R and e_+^R simplify to:

$$\alpha_+^R = \tfrac{b}{b-d} > 0 \qquad \beta_+^R = -\tfrac{b}{b-d} < 0 \qquad e_+^R = \tfrac{1}{2}b$$

The derivatives with respect to b and d are:

$$\begin{array}{cc} \frac{\partial e_+^R}{\partial b} = \frac{1}{2} > 0 & \frac{\partial e_+^R}{\partial d} = 0 \\ \frac{\partial \alpha_+^R}{\partial b} = -\frac{d}{(b-d)^2} < 0 & \frac{\partial \alpha_+^R}{\partial d} = \frac{b}{(b-d)^2} > 0 \\ \frac{\partial \beta_+^R}{\partial b} = \frac{d}{(b-d)^2} > 0 & \frac{\partial \beta_+^R}{\partial d} = -\frac{b}{(b-d)^2} < 0 \end{array}$$

Proof $\frac{\partial \alpha^R}{\partial \rho}$ can be Non-Monotonic

When there is a negative effort externality the partial derivative of α^R with respect to ρ is:

$$\frac{\partial \alpha^R}{\partial \rho} = \frac{b\left(c\left(c^2 - b^2\right) + 2R\sigma^2\left(c + 2b\rho + c\rho^2\right)\right)}{\left(b^2 + 2bc\rho + c^2 + 2R\sigma^2\left(1 - \rho^2\right)\right)^2}$$

To identify this partial derivative's sign note the numerator can be expressed as a quadratic in ρ:

$$Q = 2Rc\sigma^2\rho^2 + 4Rb\sigma^2\rho + 2Rc\sigma^2 - c\left(b^2 - c^2\right)$$

By inspection, Q is convex in ρ. The roots of $Q = 0$ are:

$$\text{Root A: } \rho = \tfrac{1}{2Rc\sigma^2}\left(\sqrt{2R\sigma^2\left(b^2 - c^2\right)\left(2R\sigma^2 + c^2\right)} - 2Rb\sigma^2\right)$$
$$\text{Root B: } \rho = -\tfrac{1}{2Rc\sigma^2}\left(\sqrt{2R\sigma^2\left(b^2 - c^2\right)\left(2R\sigma^2 + c^2\right)} + 2Rb\sigma^2\right)$$

Both roots are real from the starting assumption $b > c$.

Now consider whether these roots fall within the range $\rho \in (0, 1)$. Root B always occurs when $\rho < 0$. For Root A to be positive requires:

$$b^2 > c^2 + 2R\sigma^2$$

and for Root A to be less than one requires:

$$b < \frac{4R\sigma^2 + c^2}{c}$$

As a result, Root A occurs in the range $\rho \in (0, 1)$ if:

$$c^2 + 2R\sigma^2 < b^2 < \left(\frac{c^2 + 4R\sigma^2}{c}\right)^2 \tag{1.33}$$

It is straightforward to show this is a non-empty set of b.

Since Root B is always negative, Root A must be the second root. Combining this with Q being convex means that, if Root A occurs within the range $\rho \in (0, 1)$, to the left of Root A $\frac{\partial \alpha^R}{\partial \rho} < 0$ and to the right $\frac{\partial \alpha^R}{\partial \rho} > 0$. As such, when (1.33) holds the relationship between α^R and ρ is non-monotonic.

Partial Derivatives when $\rho \in [-1, 0)$

For brevity, a full description of the partial derivatives when $\rho = -1$ is not included. However, below, is a summary of the partial derivatives' signs when firms select their contract weights independently.

Summary - Negative Effort Externality

e - **RPE**	$\frac{\partial e^R}{\partial b} > 0$	$\frac{\partial e^R}{\partial c} \gtrless 0$	$\frac{\partial e^R}{\partial \rho} \gtrless 0$	$\frac{\partial e^R}{\partial R\sigma^2} < 0$
α - **RPE**	$\frac{\partial \alpha^R}{\partial b} \gtrless 0$	$\frac{\partial \alpha^R}{\partial c} \gtrless 0$	$\frac{\partial \alpha^R}{\partial \rho} \gtrless 0$	$\frac{\partial \alpha^R}{\partial R\sigma^2} < 0$
α - **RPE**	$\frac{\partial \alpha^R}{\partial b} \gtrless 0$	$\frac{\partial \alpha^R}{\partial c} \gtrless 0$	$\frac{\partial \alpha^R}{\partial \rho} < 0$	$\frac{\partial \alpha^R}{\partial R\sigma^2} \gtrless 0$

Summary - Positive Effort Externality

e - **RPE**	$\frac{\partial e_+^R}{\partial b} > 0$	$\frac{\partial e_+^R}{\partial d} > 0$	$\frac{\partial e_+^R}{\partial \rho} < 0$	$\frac{\partial e_+^R}{\partial R\sigma^2} > 0$
α - **RPE**	$\frac{\partial \alpha_+^R}{\partial b} \gtrless 0$	$\frac{\partial \alpha_+^R}{\partial d} \gtrless 0$	$\frac{\partial \alpha_+^R}{\partial \rho} \gtrless 0$	$\frac{\partial \alpha_+^R}{\partial R\sigma^2} < 0$
α - **RPE**	$\frac{\partial \alpha_+^R}{\partial b} \gtrless 0$	$\frac{\partial \alpha_+^R}{\partial d} \gtrless 0$	$\frac{\partial \alpha_+^R}{\partial \rho} < 0$	$\frac{\partial \alpha_+^R}{\partial R\sigma^2} < 0$

Also, when $\rho \in (-1, 0)$ it is still possible for non-monotonic relationships to exist between the contract weights and various parameters. For example, $\frac{\partial \alpha^R}{\partial \rho}$ continues to be non-monotonic when $\rho \in (-1, 0)$. The proof is very similar to that when $\rho \in (0, 1)$. The numerator of $\frac{\partial \alpha^R}{\partial \rho}$ remains a convex quadratic in ρ. Using the expressions for Root A and Root B (given earlier in this appendix) it is possible to show that Root B always occurs in the region $\rho < -1$ and so can be ignored. Similarly, it can be shown that Root A always occurs in the region $\rho > -1$. Root A will occur in the region $\rho \in (-1, 0)$ if $b^2 < c^2 + R\sigma^2$. Since Root A is the second root and the numerator of $\frac{\partial \alpha^R}{\partial \rho}$ remains a convex quadratic in ρ, to the left of Root A $\frac{\partial \alpha^R}{\partial \rho} < 0$ and to the right of Root A $\frac{\partial \alpha^R}{\partial \rho} > 0$.

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CHAPTER 2

Moral Hazard, Optimal Contracting and Strategic Competition

2.1. Introduction

This chapter demonstrates the potentially large impact a classic moral hazard problem within firms can have on equilibrium outcomes in a market involving strategic competition between firms. The chapter demonstrates that, even when companies may write non-linear contracts, the costs of agency can have a significant downward impact on equilibrium output. Indeed, as markets grow "large", in a linear demand model, the downward impact of moral hazard on expected output is greater than if firms colluded to restrict output. The importance of this result is that the welfare losses associated with collusion are seen as justifying the policy responses of antitrust authorities. The key question, therefore, is whether there is a role for policy regarding agency costs.

Some level of agency costs will often be inherent in a production process. These agency costs may arise from unobservable effort and the exposure of agents to risk or from the payment of information rents to induce truthful revelation of information by agents. Significantly, it is shown that profit-maximising firms may fail to make investments that reduce agency costs despite such investments being welfare-enhancing. As such, from a welfare perspective, there may be an over-reliance on incentive contracts. Whilst there is potential for a social planner to raise welfare, the open question is whether policymakers/regulators can actually make welfare-enhancing interventions to overcome this under-investment issue.

The link from agent effort to consumer surplus, via expected output, also shows why consumers and policymakers, not just shareholders, have a legitimate interest in the way firms resolve agency problems. Hence, this chapter highlights the importance of modelling firms as collections of utility-maximising individuals, bound together by contracts, when considering the outcomes of product market oligopolies.

A myriad of different agency problems face firms and their seriousness, particularly for large corporations, has long been recognised.[1] This chapter focuses on, arguably, the classic agency problem: unobservable effort by a risk-averse agent.[2] In the current model, agent effort alters the probability distribution of a firm's output with higher effort increasing the likelihood of higher output. Each firm comprises a single principal and a single agent. The agent, therefore, is best interpreted as a senior production manager whose effort alters the entire firm's output.

Since the agent selects their effort based on the incentive contract offered, the principal can alter the firm's expected output by altering the incentive contract. To the best of my knowledge this is the first model of agency costs in oligopoly where a cost-minimising non-linear incentive contract, incorporating effort and a performance measure as continuous variables, is derived using the first-order approach.[3] In a

[1]See Jensen and Meckling (1976).
[2]See Stiglitz (1974).
[3]Note that, in contrast to Chapter 1, RPE is not considered. By restricting attention to APE, incentive contracts involving more complex functional forms can be analysed.

generalised n-firm setting, conditions are obtained for the existence of an equilibrium in the principals' contract parameter choice game. In a parameterised setting, the model is solved with both linear and exponential inverse demand curves. Using a linear inverse demand curve, the results regarding market size are obtained. Using an exponential inverse demand curve, allows analysis regarding the number of firms and the elasticity of inverse demand.

A short literature review follows in section 2.2. In section 2.3 the model is introduced and in section 2.4 this model is solved. Section 2.5 analyses the model numerically, including for several extensions. Section 2.6 discusses the results, before section 2.7 concludes.

2.2. Literature Review

Starting with Leibenstein's (1966) discussion of "X-inefficiency", it has been widely theorised that competition between firms can alter the extent of agency problems within firms. However, research investigating the impact of agency problems within the firm on market outcomes in oligopoly is more limited. This is despite the prevalence of agency relationships within companies, the considerable attention given by researchers and policymakers to competition, and the continuing debates about executive pay. Examples of work looking at the impact of competition on agency problems include Hart (1983), Scharfstein (1988) and Schmidt (1997). Hart (1983) and Scharfstein (1988) both analyse the impact a fringe of "entrepreneurial" firms, without agency problems, has on firms that have a separation of ownership and control.

Schmidt (1997) emphasises that greater competition has a disciplining effect on employees by increasing the threat of liquidation and unemployment. However, when taken as a whole, this literature provides mixed results regarding the impact of increased competition on effort incentives.

Closer to the current paper is Raith (2003). Raith constructs a circular-city model of monopolistic competition which includes entry by firms. Here, agent effort results in reduced marginal costs. However, Raith restricts attention to linear incentive contracts, and the paper's aim is different from the current investigation. Raith's aim is to create a setting where a positive relationship between risk and incentive strength exists.[4]

As discussed in Chapter 1, the strategic delegation literature does investigate the impact of incentive contracts on product market competition.[5] However, whilst this literature is described in terms of a "principal" and an "agent", it generally assumes agents are risk-neutral and so agency costs are ignored. Instead, the role of delegation itself as a strategic commitment device is emphasised. Whilst accepting that incentive contracts can be used strategically, the current chapter, like Chapter 1, takes the original theoretical basis for incentive contracts - to induce effort - as the primary motivation for their existence.

Gal-Or (1997) provides an overview of the small number of contributions to the strategic delegation literature that do include true agency problems. However, in these

[4]Other papers looking at the impact of competition on incentives include Holden (2008), Plehn-Dujowich and Serfes (2010) and Theilen (2009).
[5]Early works include Vickers (1985), Fershtman and Judd (1987) and Sklivas (1987).

papers, such as Fumas (1992) and Gal-Or (1993), attention is again restricted either to linear incentive contracts or to decisions regarding delegation and organisational structure. The salesforce compensation literature, for example Bhardwaj (2001), also looks at delegation questions whilst incorporating moral hazard problems. In this literature, Mishra and Prasad (2005), unusually, do consider optimal non-linear incentive contracts. However, Mishra and Prasad only demonstrate the combinations of centralised pricing and delegation that can be equilibrium candidates, rather than fully deriving the equilibrium incentive contracts.

Moving away from delegation issues, Hermalin (1994) demonstrates that non-convexities in firms' agency problems can cause otherwise identical firms to offer incentives of differing strengths. Looking at agency issues from a different perspective, Aghion, Dewatripont and Rey (1999) consider the role of external financing as a disciplining device on managers of firms engaged in oligopolistic competition. Lastly, Bonatti (2003) suggests the effort-monitoring capabilities of unions may favour the use of collective bargaining in oligopoly settings.

2.3. The Model

This section describes the parameterised model's structure. The model is presented from the perspective of firm (principal-agent pair) i.

Two firms compete in a quantity competition game. The overall game involves three stages:

Stage 1 - Each principal derives the optimal incentive contract for their agent. This includes selecting the optimal effort level to induce.

Stage 2 - Given the incentive contract offered, each agent selects the effort level maximising their utility.

Stage 3 - The outcome of the production process is realised. The output is sold at the price required to clear the market. All players receive their pay-offs.

In each stage the players take decisions simultaneously and independently. The principal-agent pairs are assumed identical in all respects.[6] Output is homogeneous and the output of each firm is a random variable dependent on agent effort. Firm i's output is denoted q_i, where $q_i \in [0, \infty)$ and $Q = q_i + q_j$. Agent i's effort is denoted a_i, where $a_i \in [\underline{a}, \overline{a}]$, $\underline{a} > 0$ and $A = a_i + a_j$.[7] The inverse demand function is linear:

$$P(Q) = \left\{ \begin{array}{cc} B - Q, & B \geq Q \\ 0, & B < Q \end{array} \right\}$$

Output is exponentially distributed with the probability density function for output, given a specific effort level a_i, being:

$$f(q_i|a_i) = \left\{ \begin{array}{cc} \frac{1}{a_i} e^{-\frac{q_i}{a_i}}, & q_i \geq 0 \\ 0, & q_i < 0 \end{array} \right\}$$

[6]The proof for the existence of equilibrium, shown in Appendix 2.1, does not require symmetry.
[7]Assume that $\underline{a}$ is sufficiently low, and $\overline{a}$ is sufficiently high, that they never impinge on the equilibrium outcome.

The exponential distribution is used primarily for tractibility. It is also an example of a distribution offering an number of important features. Firstly, the exponential distribution's support is $[0, \infty)$ thereby ruling out negative quantities. Secondly, since the distribution's support remains constant regardless of effort, the principal can never use output to infer the agent's effort with certainty. Thirdly, it ensures Jewitt's (1988) conditions for the first-order approach to be valid hold and, lastly, the cumulative distribution for output, $F(q_i|a_i)$, is such that a distribution where a_i is high will display first-order stochastic dominance relative to a case where a_i is low. The condition for this last property to hold is that $F_{a_i}(q_i|a_i) \leq 0$ holds for all q_i and $F_{a_i}(q_i \mid a_i) < 0$ holds for some q_i.[8] As such, output is an informative signal of effort.

Each principal is risk-neutral and principal i's utility function is $Y(\pi_i) = \pi_i$ where π_i is the principal's expected pay-off. Agent i's utility is denoted $U(w_i)$, where w_i is agent i's income.[9] Since the agent is risk-averse, $U(w_i)$ is increasing concave in w_i. The agent's total utility is denoted $\Omega\left(w_{i,}a_i\right)$ and the disutility from effort is denoted $V(a_i)$. The agent's utility from income and disutility from effort are additively separable so that $\Omega\left(w_{i,}a_i\right) = U(w_i) - V(a_i)$. In the model solved below $V(a_i) = a_i^2$ and $U(w_i) = 2\left(w_i\right)^{\frac{1}{2}}$.[10]

As effort is unobservable, assume the principal must use an incentive compatible contract that is a continuous function of output to induce effort. Denote this contract $w_i(q_i)$. As the output distributions for each firm are independent, comparing q_i and q_j via relative performance evaluation does not reduce costs. Similar reasoning also means there is no benefit from rewarding an agent on the basis of own-firm profits.

8 The exponential distribution actually possesses the even stronger Monotone Likelihood Ratio Property.

9 For simplicity, assume the agent has no other wealth or income sources.

10 This particular utility function is used for tractibility.

Assume the labour market is competitive and denote the agent's reservation utility $R \geq 0$. Assume there are no other production costs beyond the cost of labour. Also, assume each principal must always operate their firm, employ an agent and induce the minimum effort level $\underline{a}$.[11]

Beyond the effort exerted, there is no other hidden information in the model. Assume each principal knows the shape of their own agent's utility function and all the characteristics of the rival principal-agent pair.

2.4. Solving the model

The objective when solving the model is to obtain a pure strategy Nash equilibrium for the contract parameters, also denoted a_i (a_j), selected by each principal.

Principal i's problem can be split into two separate steps. The first is to derive a contract which induces a given level of effort from agent i at the minimum cost. The second is to select the amount of effort to induce, via the incentive contract, to maximise profits given the product market interaction. Each of these steps is considered in turn.

Step 1: Deriving the cost-minimising contract

The incentive contract must satisfy the agent's participation constraint (PC) and incentive compatibility constraint (ICC). Formally this problem can be expressed as:

[11] This assumption ensures no downward jump in the firm's reaction function occurs which could affect the proof of existence. As such, the model considers a short-run setting.

$$\max_{w_i(q_i)} \int_0^\infty -w_i(q_i)dF(q_i|a_i) \tag{2.1}$$

subject to the PC:

$$\int_0^\infty 2\left(w_i(q_i)\right)^{\frac{1}{2}} dF(q_i|a_i) - a_i^2 \geq R \tag{2.2}$$

and the ICC:

$$\int_0^\infty 2\left(w_i(q_i)\right)^{\frac{1}{2}} dF_{a_i}(q_i|a_i) - 2a_i = 0 \tag{2.3}$$

The cost-minimising contract is found using the first-order approach as in Mirrlees (1976) and Holmstrom (1979). To be incentive compatible, the contract must maximise the agent's expected utility at the effort level the principal wishes to induce. The first-order approach uses the first-order condition (FOC) of the agent's maximisation problem as a sufficient condition for reaching the global maximum of the agent's problem. Thus, the agent's FOC is used as the ICC in the principal's maximisation problem shown above.

Mirrlees (1999)[12] established that using an agent's FOC as the ICC is not generally valid. The FOC is a necessary, rather than sufficient condition, for maximising the agent's utility. However, Jewitt (1988) provides conditions for which the FOC is a

[12] This paper was originally completed in 1975 but not published.

sufficient condition for the maximisation of an agent's utility. Jewitt's conditions on the distribution function and utility function are:

(i) $\int_{-\infty}^{q} F(q|a)dq$ is non-increasing convex in a for each value of q;

(ii) $\int qdF(q|a)dq$ is non-decreasing concave in a;

(iii) $\frac{f_a(q|a)}{f(q|a)}$ is non-decreasing concave in q for each value of a;

(iv) the utility of the agent is a concave increasing function of the observable variables; mathematically $\omega(z) = U\left(U'^{-1}\left(\frac{1}{z}\right)\right)$, where $z > 0$, is concave.

An explanation of why these conditions are needed is provided in Appendix 2.2.

Lemma 2.1 *Jewitt's (1988) conditions for the validity of the first-order approach are met by the current parameterised model.*

Proof. Jewitt states that all the distributions falling within the exponential family meet conditions (i)-(iii). Regarding condition (iv), since $\frac{1}{U'(w)} = (w(q))^{\frac{1}{2}}$, $U(w)$ is a linear, and therefore concave, transformation of $\frac{1}{U'(w)}$. Hence, using the first-order approach is valid. ■

Whilst Jewitt's conditions ensure the agent's maximisation problem is concave, they do not ensure the concavity of the principal's maximisation problem.[13] At present the strict concavity (strict quasiconcavity) of the principal's problem is assumed.

[13]This issue is noted by Grossman and Hart (1983).

Lemma 2.2 *The cost-minimising contract for principal i to induce an effort a_i is:*

$$w_i^*(q_i) = \frac{1}{4}\left(a_i^2 + R + 2a_i\left(q_i - a_i\right)\right)^2$$

Proof. Principal i's problem can be expressed as the following Lagrangian :

$$\max_{w_i(q_i)} L_i\left(a_i\right) = \int_0^\infty -w_i(q_i)dF(q_i|a_i) \tag{2.4}$$

$$+\lambda_i\left[\int_0^\infty 2\left(w_i(q_i)\right)^{\frac{1}{2}} dF(q_i|a_i) - a_i^2 - R\right]$$

$$+\mu_i\left[\int_0^\infty 2\left(w_i(q_i)\right)^{\frac{1}{2}} dF_{a_i}(q_i|a_i) - 2a_i\right]$$

In (2.2) the PC is an inequality constraint; however, in (2.4) it is assumed to bind with equality. Intuitively this assumption must be true. If the PC did not bind with equality, the principal could reduce the transfer payment made to the agent, thus strictly increasing profits, whilst still ensuring the agent accepted the contract.

The necessary condition for the cost-minimising contract is found by holding a_i fixed and taking the partial derivative of (2.4) with respect to $w_i(q_i)$. Setting this derivative equal to zero gives:

$$\frac{\partial L_i}{\partial w_i(q_i)} = -\int_0^\infty dF(q_i|a_i) + \lambda_i \int_0^\infty \left(w_i(q_i)\right)^{-\frac{1}{2}} dF(q_i|a_i)$$

$$+\mu_i \int_0^\infty \left(w_i(q_i)\right)^{-\frac{1}{2}} dF_{a_i}(q_i|a_i) = 0$$

Dividing throughout by $\left(w_i(q_i)\right)^{-\frac{1}{2}} dF(q_i|a_i)$ and recognising that $dF(q_i|a_i) = f(q_i|a_i)dq_i$ leads to:

$$\left(w_i(q_i)\right)^{\frac{1}{2}} = \lambda_i + \mu_i \frac{f_{a_i}(q_i|a_i)}{f(q_i|a_i)}$$

Inserting the expressions for $f(q_i|a_i)$ and $f_{a_i}(q_i|a_i)$ gives:

$$w_i(q_i) = \left(\lambda_i + \frac{\mu_i}{a_i^2}(q_i - a_i)\right)^2 \tag{2.5}$$

Inserting (2.5) into the ICC and the PC and then solving as a system of two equations in two unknowns gives:

$$\lambda_i = \frac{a_i^2 + R}{2}$$

$$\mu_i = a_i^3$$

Inserting these values for λ_i and μ_i back into (2.5) gives the cost-minimising contract to induce the effort a_i as:

$$w_i^*(q_i) = \frac{1}{4}\left(a_i^2 + R + 2a_i(q_i - a_i)\right)^2 \tag{2.6}$$

■

Step 2: Selecting the optimal value of a_i

The second step of the principal's problem is to select the optimal value of a_i to write in the incentive contract given by (2.6). Note a_i is both a parameter in the contract and the level of effort agent i will exert given (2.6). As such, when selecting the optimal value of a_i to write in the contract, the principal is selecting the optimal effort level to induce in the agent.

Holding principal j's choice of a_j fixed, the unconstrained profit maximisation problem facing principal i is:

$$\max_{a_i} E(\pi_i) = \int_0^B \int_0^{B-q_j} (B - q_i - q_j)\, q_i dF(q_i|a_i) dF(q_j|a_j) - \int_0^\infty w_i^*(q_i) dF(q_i|a_i)$$

Substituting in the expressions for $w_i^*(q_i)$, $dF(q_i|a_i)$ and $dF(q_j|a_j)$, setting $R = 0$[14] and integrating, gives firm i's expected profit function as:

$$E(\pi_i) = a_i a_j^3 \frac{e^{-\frac{B}{a_j}}}{(a_i - a_j)^2} + a_i^2 e^{-\frac{B}{a_i}} \frac{Ba_i - Ba_j + 2a_i^2 - 3a_i a_j}{(a_i - a_j)^2} \tag{2.7}$$

$$+ (B - 2a_i - a_j)\, a_i - \frac{5}{4} a_i^4$$

Assuming that $E(\pi_i)$ is strictly quasiconcave and there exists a point such that $\frac{\partial E(\pi_i)}{\partial a_i} = 0$ in the range of a_i considered, then the FOC, $\frac{\partial E(\pi_i)}{\partial a_i} = 0$, will be a necessary and sufficient condition for profit maximisation. The full FOC is:

$$\frac{\partial E(\pi_i)}{\partial a_i} = -a_j^3 \frac{e^{-\frac{B}{a_j}}}{(a_i - a_j)^3} (a_i + a_j) \tag{2.8}$$

$$+ \frac{e^{-\frac{B}{a_i}}}{(a_i - a_j)^3} \left(\begin{array}{c} B^2 a_i^2 - 2B^2 a_i a_j + B^2 a_j^2 + 3Ba_i^3 \\ -8Ba_i^2 a_j + 5Ba_i a_j^2 + 4a_i^4 - 11a_i^3 a_j + 9a_i^2 a_j^2 \end{array} \right)$$

$$+ B - 4a_i - a_j - 5a_i^3 = 0$$

An equivalent condition exists for principal j.

The next step is to demonstrate an equilibrium exists in the principals' contract parameter choice game.

[14] $R = 0$ is imposed for simplicity.

Theorem 2.1 *In an n-firm setting, if:*

(i) Jewitt's (1988) conditions for the validity of the first-order approach hold, and

(ii) each principal's profit function is strictly quasiconcave in their own contract parameter (a_i for principal i),

then an equilibrium will exist in the principals' contract parameter choice game.

Proof. The proof is adapted from the proof of existence of an n-firm Cournot equilibrium by Frank and Quandt (1963). The proof is shown in Appendix 2.1.[15]

Given Theorem 1, the equilibrium values of a_i and a_j can be found numerically using (2.8) and the equivalent condition for principal j.

2.5. Numerical Analysis

2.5.1. Varying market size (B)

To provide comparative benchmarks for the impact of moral hazard on market outcomes, two additional scenarios are considered. The first is where effort is observable

[15]If the non-standard demand function:

$$P(q_i, q_j) = B - q_i - q_j \text{ for all } q_i \geq 0, q_j \geq 0$$

is used a unique equilibrium exists in the principals' contract parameter choice game for the parameterised version of the model. The proof for this is an adaption of Szidarovsky and Yakowitz's (1977) proof of an unique Cournot equilibrium. Imposing additional assumptions, this proof can be extended to a general n-firm setting.
Significantly, in a parameterised model using this non-standard demand function, it is possible to prove that each principal's profit maximisation problem is concave. It is also possible to show that, as B grows large, the equilibrium values of a_i and a_j found using this non-standard demand function tend to those using the standard demand function stated in section 2.3. The workings for these results are available on request.
This alternative inverse demand function is not used as it allows negative prices to occur at high output levels. As such, increasing the variance of output is particularly costly in this alternative setting. Compared to the case where the inverse demand curve includes the constraint $P \geq 0$, the equilibrium output/effort induced is lower. One paper which introduces random quantities into a Cournot model, Deo and Corbett (2009), uses this non-standard model allowing negative prices. However, Deo and Corbett do not incorporate a principal-agent problem into their model.

and verifiable, thus removing the agency problem. This scenario is referred to as the "First Best". In the second scenario, effort is again observable and verifiable but now firms act to maximise joint profits, i.e. collude.[16] This second benchmark is chosen as the lost output/welfare resulting from collusion is seen as sufficient to justify antitrust laws.

Note, for the exponential distribution, a firm's expected output equals the effort exerted by the firm's agent, i.e. $E(q_i) = a_i$.

To solve the problem numerically, appeal to the problem's symmetry and consider a symmetric equilibrium such that $a_i^* = a_j^* = a$. Using this fact, setting $R = 0$ and applying l'Hôpital's rule three times to (2.8) gives the equilibrium condition for moral hazard. This condition, along with the conditions, for the other two scenarios, are stated in the table below. The other two conditions are derived in Appendix 2.3.

First Best	$\frac{1}{a^2}e^{-\frac{B}{a}}\left(\frac{1}{3}B^3 + \frac{3}{2}B^2a + 4Ba^2 + 5a^3\right) + B - 5a - a^3 = 0$
Moral Hazard	$\frac{1}{a^2}e^{-\frac{B}{a}}\left(\frac{1}{3}B^3 + \frac{3}{2}B^2a + 4Ba^2 + 5a^3\right) + B - 5a - 5a^3 = 0$ [17]
Maximisation of Joint Profits	$\frac{1}{2a^2}e^{-\frac{B}{a}}\left(B^3 + 4B^2a + 10Ba^2 + 12a^3\right) + B - 6a - a^3 = 0$

Result 2.1 *As the market becomes "large", moral hazard has a far greater downward impact on expected output per firm than maximisation of joint profits (collusion) by firms.*[18]

[16]The ability to sustain collusion is assumed rather than demonstrated.

[17]Given that expected wage costs remain strictly convex in a_i when there is observable and verifiable effort, Theorem 2.1 also holds for the cases of the first best and maximisation of joint profits.

[18]Obtaining this result analytically from the equilibrium conditions has been attempted using the implicit function theorem. However, the resulting expressions for $\frac{\partial a}{\partial B}$ and $\frac{\partial^2 a}{\partial B^2}$ are too complex to be signed unambiguously.

This central result is shown in Figure 2.1.[19] In the top graph, expected output per firm is plotted for values of B from 0.05 to 5. This highlights that for "small" market sizes, collusion causes a greater reduction in expected output than moral hazard. The bottom graph considers values of B from 0.05 to 250. It illustrates moral hazard's greater downward impact on expected output as the market becomes "large".[20] Indeed, when $B = 250$, expected industry output when moral hazard is present is 41% beneath the expected industry output when collusion occurs.

The relative importance of moral hazard versus collusion varies according to the relative size of agency costs compared to the negative revenue externalities associated with competition.[21] The relative importance of moral hazard and collusion changes according to the specifications of the demand curve, the cost of effort function and the utility function. The cost of effort and utility functions together determine the agency cost. The demand curve determines the size of the negative revenue externalities, when firms operate independently. These externalities are internalised when firms collude to maximise their joint profits. The larger the externalities to be internalised, the greater the drop in expected output compared to the "First Best" when firms maximise joint profits.

[19]The MATLAB M-files generating Figures 2.1 and 2.3-2.6 are available on request.

[20]If the agents' reservation utility, R, is high enough, then for all values of B large enough to generate positive expected profits moral hazard will have a greater downward impact on expected output than maximisation of joint profits.

[21]When setting their own contract parameter, independent firms fail to consider the impact their choice has on the revenue received by rival firms. If firm i increases its contract parameter, it increases firm i's expected output and, hence, lowers the expected price for firm j's output.

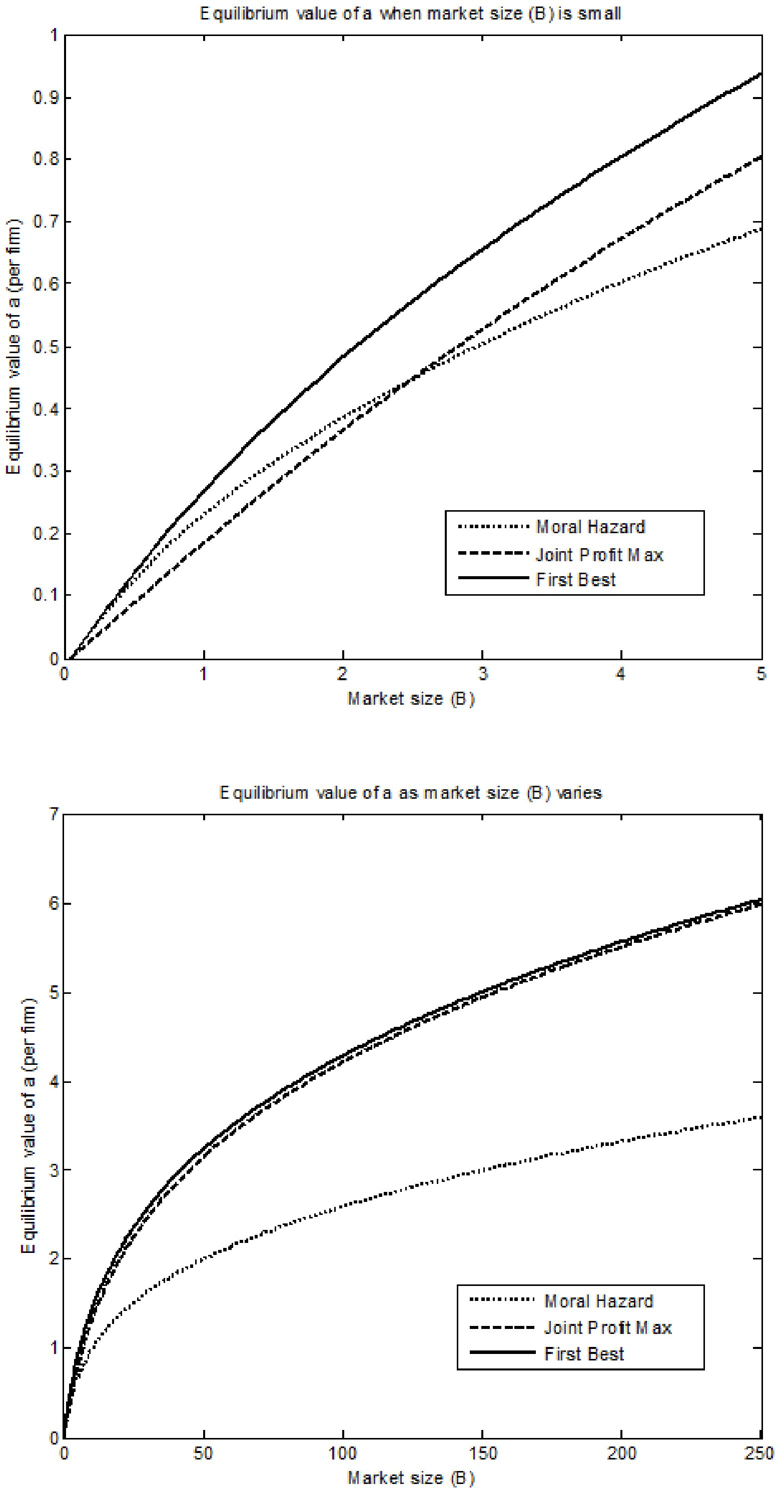

Figure 2.1: Equilibrium expected output per firm as market size, B, increases.

In the current structure, the agency cost (the difference in wage costs when moral hazard is, and is not, present) is a_i^4 and so is convex in effort. Assuming B is large relative to a_i, the externality firm i causes is approximately $-a_i a_j$.[22] Therefore, when market size increases and firms induce more effort, the agency cost increases at a faster rate than the externality. Hence, when B is large, a firm facing a moral hazard problem will set a lower value of a than one maximising joint profits.

2.5.2. Extension 1 - Investment in a perfect monitoring technology

The central question is whether the results above have policy implications. There is a potential role for policy only if a benevolent social planner could increase welfare. If agency costs are an inherent part of the production process, there will be no role for policy. However, it is probable that firms do have means to reduce agency costs. For example, firms could invest in productivity-monitoring software or employ "mystery shoppers". The issue is whether the incentives for profit-maximising firms to minimise agency costs lead to welfare-maximising outcomes. It is already known, from Shleifer and Vishny (1986), that if ownership of a firm is dispersed, i.e. there are multiple principals, then there may be under-investment in monitoring. This is because each principal can free-ride on the monitoring effort of others. This section highlights that under-investment in monitoring may occur for another reason: firms do not consider the gains in consumer surplus associated with reduced agency costs.

[22] Recall from footnote 15 that when B is large the current model can be approximated with one where the inverse demand curve is: $P(q_i, q_j) = B - q_i - q_j$ for all $q_i, q_j \geq 0$. The value $-a_i a_j$ comes from this alternative model.

For simplicity, assume firms can invest in a perfect monitoring technology that makes effort observable and verifiable. Let each firm have a discrete action set $D \in \{Invest, NotInvest\}$. Simultaneously, each firm decides which action to play from D in a stage game prior to the game laid out in section 2.3. The cost of the investment to each firm is C.

Three numerical examples are considered where B is set to 50, 100 and 200 respectively. The aim is to see if the range of C where profit-maximising firms play $Invest$ matches the range of C where a social planner, maximising total surplus, would choose $Invest$.

Result 2.2 *There exists a range of investment costs such that profit-maximising firms will not undertake an investment in a perfect monitoring technology, despite it being welfare-enhancing. As market size, B, increases so does the region of investment costs where private firms and a benevolent social planner make different investment decisions.*

Proof: See Result A2.1 in Appendix 2.4

The divergence in investment decisions is summarised in Figure 2.2. Where there is a divergence, one can say that profit-maximising firms will have an over-reliance on incentive contracts from a welfare perspective.

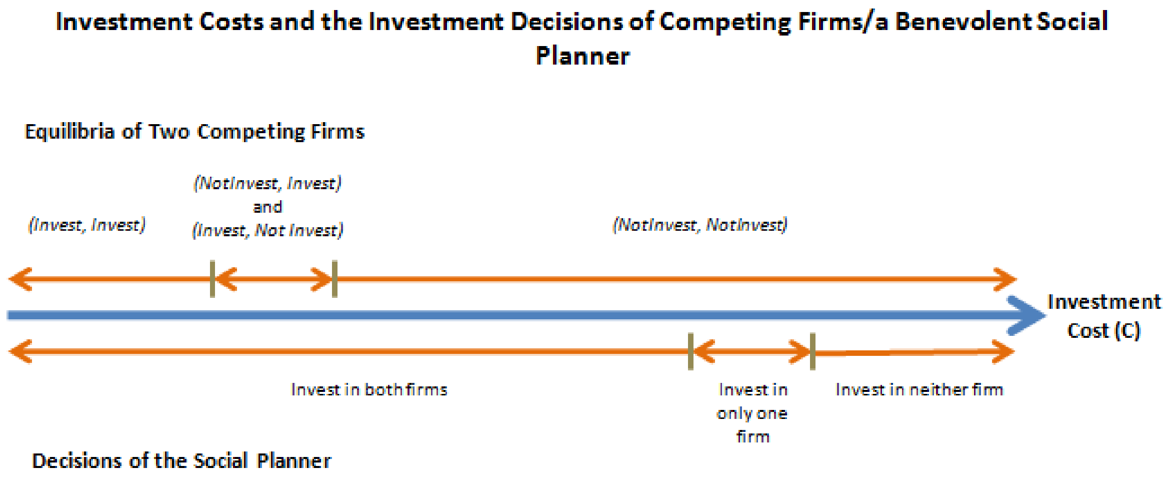

Figure 2.2: The differing investment decisions of independent firms and a benevolent social planner.

The size of the region where the investment choices of firms and a social planner differ will depend on the model's specification. However, the underlying intuition for this divergence appears robust. Unless firms can fully capture their products' consumer surplus they will always be less willing to invest than a welfare-maximising social planner. As the market becomes larger, i.e. B has a higher value, the difference between the equilibrium expected output under moral hazard and the first best becomes larger (see Figure 2.1). That this difference increases, along with the marginal willingness to pay of consumers, means that the difference between the consumer surplus under moral hazard and the first best also increases with market size. Hence, the investment cost range where the investment decisions of private firms and a benevolent social planner diverge becomes larger as market size, B, is increased (see Result A2.1).

2.5.3. Extension 2 - An exponential inverse demand curve

Whilst using a linear inverse demand curve provides consistency with basic oligopoly models, the resulting expected revenue function significantly complicates analysis. The analysis is simplified when, instead, an exponential inverse demand curve is used. Significantly, this specification change makes it possible to prove that the expected profit function is strictly quasiconcave in the parameterised setting. Also, it allows analysis regarding the number of firms and the elasticity of the inverse demand curve.

The inverse demand curve used in this extension is:

$$P(Q) = De^{-\tau Q} \tag{2.9}$$

where D and τ are strictly positive constants and Q is industry output. D represents the highest willingness to pay of any consumer (i.e. the vertical intercept of the inverse demand curve) whilst τ affects the elasticity of the inverse demand curve. The elasticity of price with respect to quantity is given by:

$$\varepsilon = \frac{dP}{dQ}\frac{Q}{P} = -\tau Q$$

An increase in τ increases this elasticity's absolute value thus making price more sensitive to output changes.

Lemma 2.3 *For the inverse demand function:*

$$P(Q) = De^{-\tau Q}$$

the expected profit function is strictly quasiconcave in a_i if the assumptions in section 2.4 hold. Hence, an n-firm equilibrium in the contract parameter choice game exists. For the two-firm case the equilibrium is unique.

Proof. Theorem 2.1 again applies so an n-firm equilibrium exists in a generalised setting for the exponential inverse demand curve in (2.9) if the assumptions in section 2.4 hold. Apart from changing the expected revenue function in (2.13), there are no other changes to the proof of Theorem 2.1. The proofs establishing $E(\pi_i)$ is strictly quasiconcave and that the two-firm equilibrium is unique are provided in Appendix 2.5.

This variation in equilibrium expected output as the parameters D, τ and n are varied is now analysed numerically. Again the three scenarios of the first best, moral hazard and joint profit maximisation are considered. The expected profit functions and the equilibrium conditions are stated in Appendix 2.5.

Result 2.3 *As D, τ and the number of firms are increased, the relative importance of moral hazard compared to joint profit maximisation decreases.*

Firstly, consider the impact of D and τ on the equilibrium value of a for the two-firm case. Set $R = 0$. When D is varied let $\tau = 0.1$. When τ is varied let $D = 100$. The results, for total industry output, $2a$, are shown in Figure 2.3.

As D and τ are varied, whether moral hazard or joint profit maximisation has the greater downward impact on expected output also varies. When D and τ are small, moral hazard has the greater downward impact. As D and τ grow large, joint profit maximisation has the greater downward impact. This switch results from the changing relative size of agency costs against the negative revenue externalities of competition. Increasing D causes the inverse demand curve's slope to become increasingly negative:

$$\frac{\partial P\left(Q\right)}{\partial Q} = -\tau D e^{-\tau Q}$$

When D is high, a given increase in output will cause a greater absolute drop in price and, hence, a larger revenue externality. When τ increases, the argument is similar except that an increase in τ now leads to a greater percentage drop in price for a given percentage increase in output.[23]

Lastly, the impact of changing the number of firms, n, is considered. Numerical analysis equivalent to that for the two-firm case has been performed for the three- and four-firm cases. The results are shown in Figures 2.4 and 2.5.

[23] Also, note that, as D and τ grow large, the equilibrium values of a under moral hazard and in the "First Best" converge. Whilst this result might appear odd, note that for increases in τ the equilibrium value of a falls. As such, when τ is increased, the agency cost's absolute size falls. Hence, moral hazard has a smaller downward impact on expected output than maximisation of joint profits. For increases in D, the convergence is understandable given the FOC for profit maximisation. Firm i's FOC when moral hazard is present is:

$$\frac{\partial E(\pi_i)}{\partial a_i} = \frac{D(1-\tau a_i)}{(\tau a_i+1)^3(\tau a_j+1)} - 5a_i^3 = 0$$

(See Appendix 2.5 for the derivation). If $a_i \geq \frac{1}{\tau}$, so that marginal revenue is negative and the firm is on the inelastic section of its demand curve, this condition can never hold. As such, there is an upper limit to the equilibrium value of a_i regardless of the value of D.
In the "First Best" this upper limit on a_i is reached at a lower value of D than when there is moral hazard.

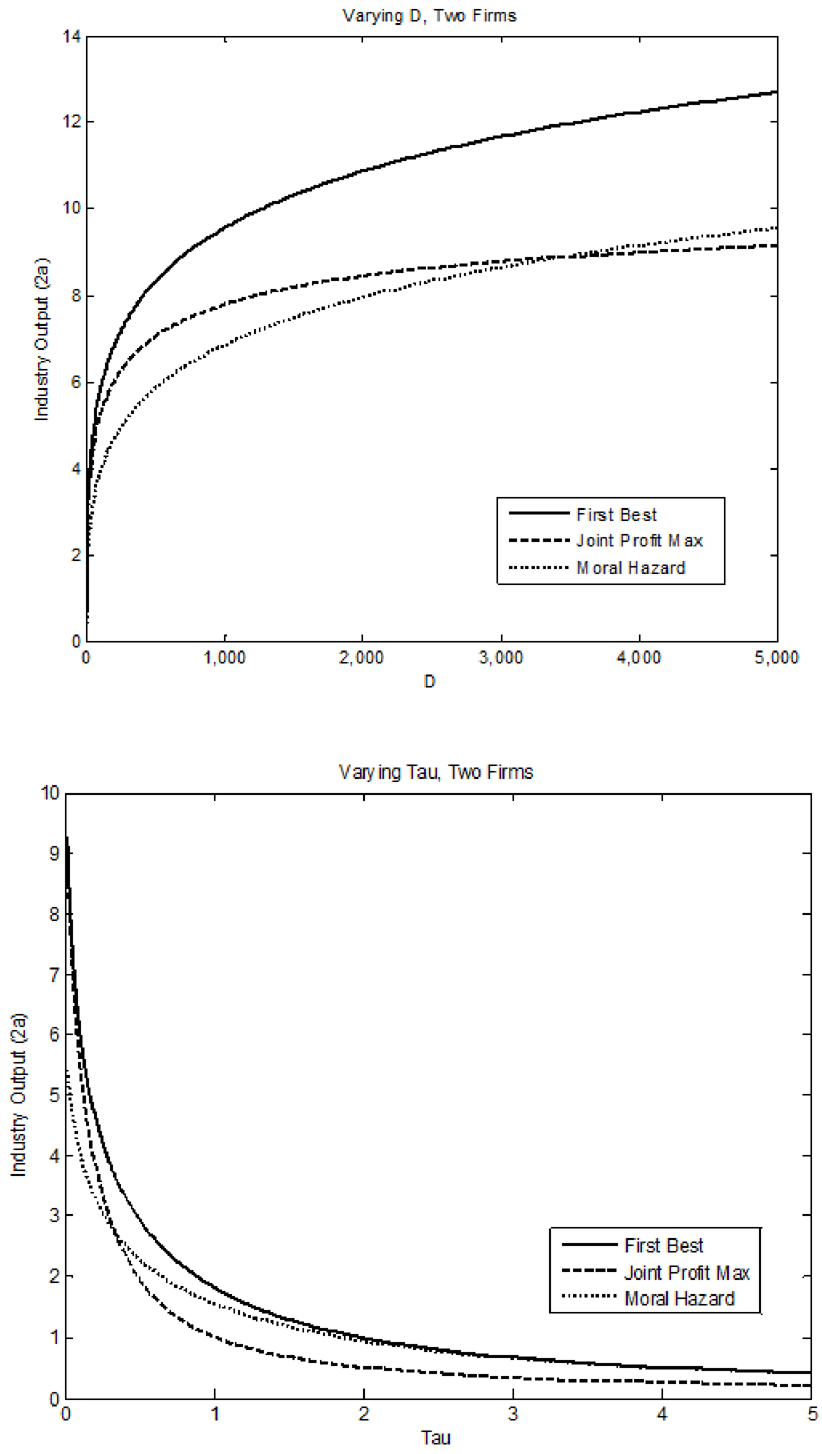

Figure 2.3: Expected industry output in a two-firm industry as D and τ are varied.

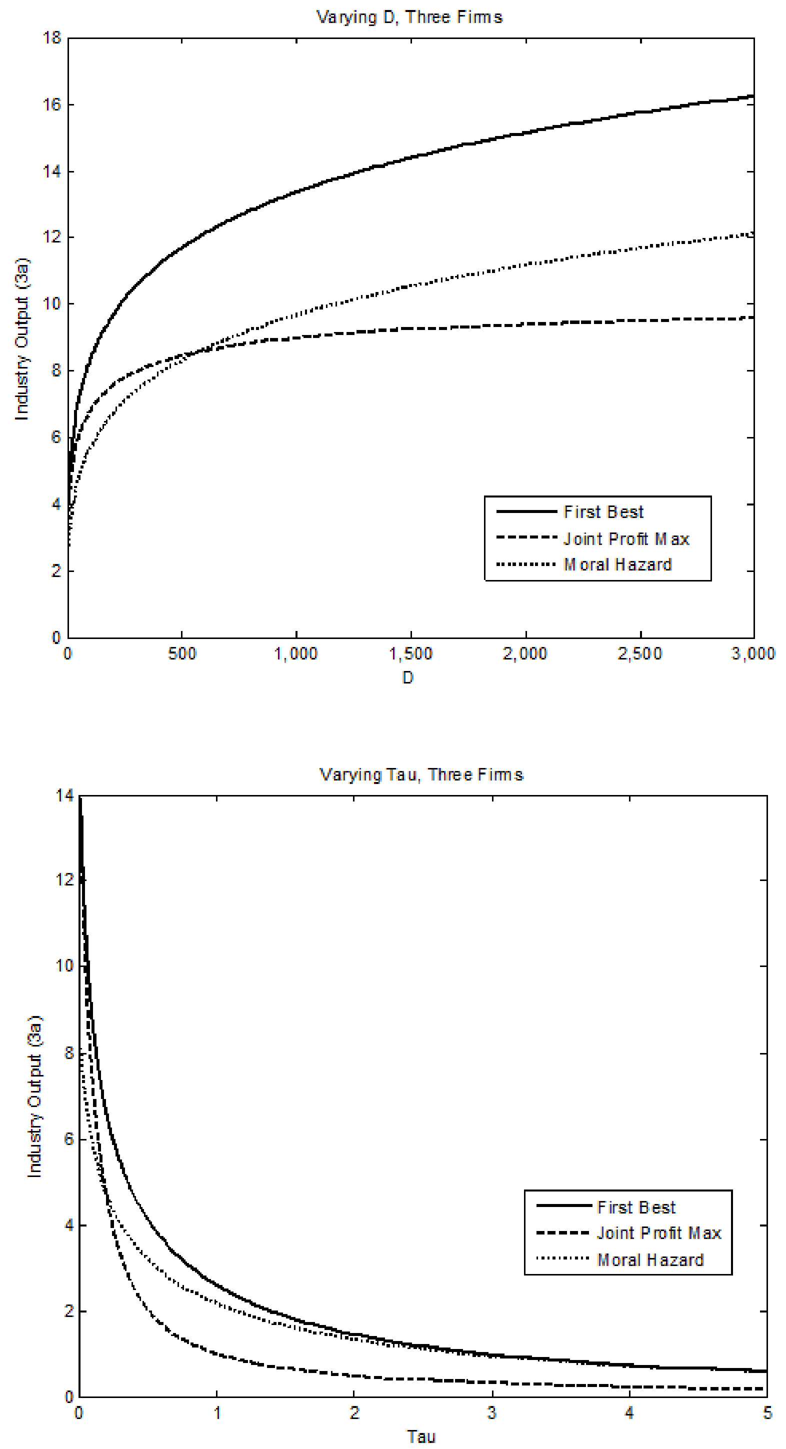

Figure 2.4: Expected industry output in a three-firm industry as D and τ are varied.

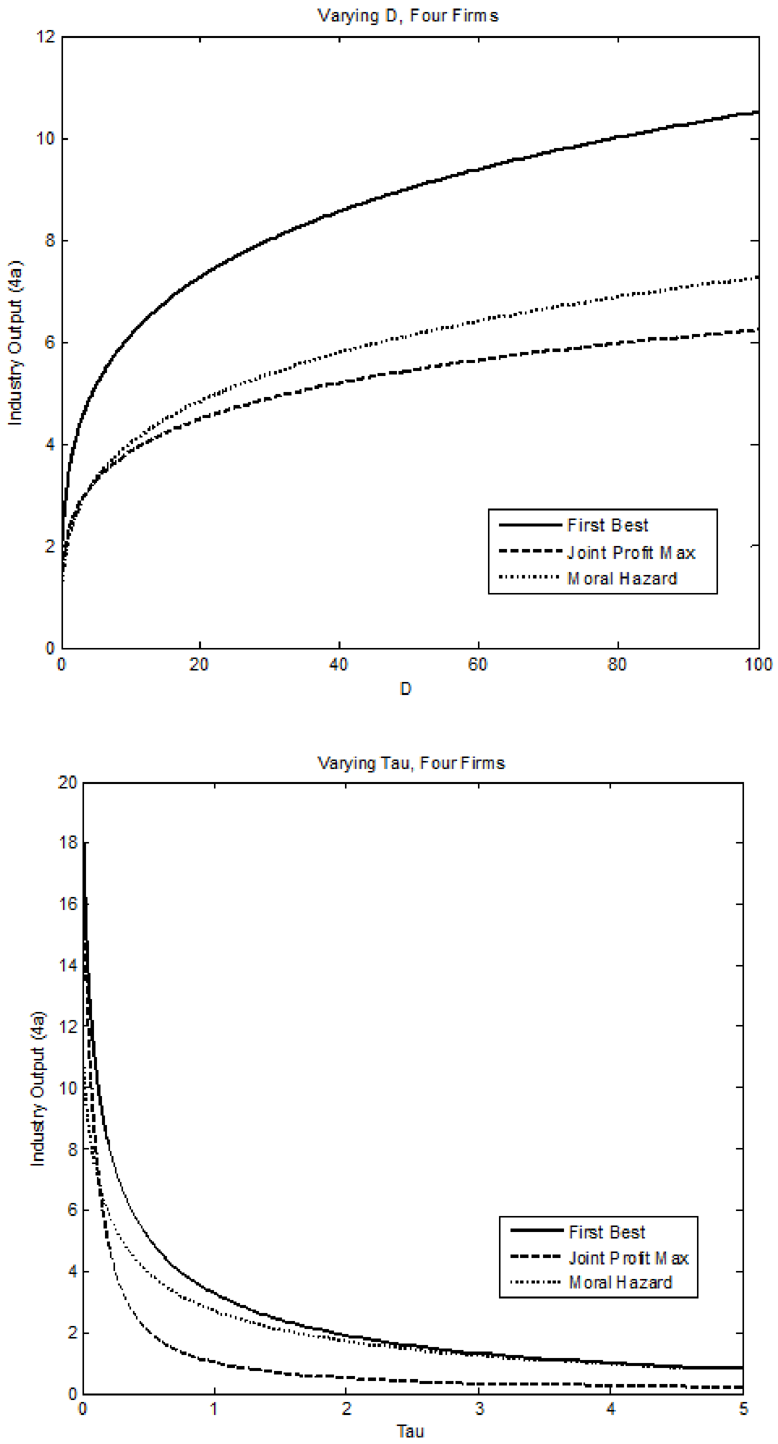

Figure 2.5: Expected industry output in a four-firm industry as D and τ are varied.

As n increases so does expected industry output. The figures also show that, as n increases, the critical values of D and τ at which collusion (rather than moral hazard) has the greater downward impact on expected output fall. The values of D and τ where the crossing points occur are:

Number of Firms	**Value of** D $(\tau = 0.1)$	**Value of** τ $(D = 100)$
2	3439.5	0.325
3	573.6	0.179
4	186.0	0.123

That the values of D and τ where the crossing points occur are decreasing in n is unsurprising for two reasons. Firstly, as n increases the negative revenue externalities when firms compete increase. As such, when joint profit maximisation internalises these externalities, there is a greater drop in expected industry output. Secondly, as n increases, the effort each firm induces in its agent falls as each firm has a smaller market share. Since agency costs are increasing in effort, the costs of agency are reduced and so the downward impact of moral hazard on expected output is reduced. Intuitively, it seems likely that this relative drop in the significance of moral hazard as n increases is robust to the demand curve's specification.

2.6. Robustness and Discussion

Whilst the main results have been obtained in a specific parameterised model, the basic intuitions behind the results seem to apply more generally. Whether moral hazard or joint profit maximisation has a greater downward impact on expected output, will depend on the relative sizes of the agency cost and the negative revenue externalities between firms. The intuition that firms may not invest in welfare-enhancing agency cost reductions, if they cannot fully capture the resulting increases in consumer surplus, also appears robust.

The magnitude of agency costs in this model is clearly influenced by the assumption of each firm employing a single agent. When addressing questions relating to senior executives, this assumption appears plausible due to inherent indivisibilities in senior executives' tasks. Nevertheless, incorporating multiple agents in each firm represents an important future extension. With multiple workers the impact of moral hazard is likely to decrease. If a firm's total output is held constant and the number of workers increases, each individual worker will be required to exert less effort. Since agency costs per worker are increasing in effort, as effort per workers falls so do the costs of agency.[24]

For agency costs to remain an important determinant of market outcomes two features are probably required: (i) an upper limit on the number of agents employed by a firm and (ii) barriers to entry for firms. The combination of these constraints would limit the number of agents in an industry implying that each agent would have to exert a significant amount of effort to achieve a high expected output. To some extent, the fixed costs associated with employing additional workers may limit workforce size. Indeed, one could endogenise workforce size as an initial stage game to investigate this issue.

[24] However, environments with multiple workers may create new agency issues. For example, if output per worker is not observable, workers may free-ride on the efforts of others, as in the team production literature.

Also, it would seem natural to relax the assumption of a perfectly competitive labour market. If there was a binding constraint on labour supply, to increase expected output, the only option would be to increase effort per worker. A restriction on labour supply would also limit entry by firms. This logic suggests that agency costs due to unobservable effort may be particularly high within professions where there are a limited number of qualified individuals, such as law and medicine. In each of these professions the perception of high workloads, the need to exert high effort and, therefore, relatively high pay seems to fit the intuition of the model.

One should also recognise the possible role of the exponential distribution in driving Result 2.1. For the probability density function described in section 2.3, increasing effort increases both the expected value and the variance of output. As a result, if the principal wishes to induce additional effort, the agent must be compensated both for the additional effort exerted and for being exposed to a "riskier" performance variable. To address this issue, one could develop a model using the simpler LEN-framework[25] where effort reduces marginal costs. In the LEN-framework, the incentive contract is linear, the agent's utility function is exponential and there is a normally distributed additive shock term. Using the normal distribution ensures the performance measure has a constant variance. However, a linear contract is unlikely to be cost-minimising. If real-world contracts are not cost-minimising, the current model would represent a lower-bound on the agency costs facing firms and on the downward impact of moral hazard.[26]

[25] See Holmstrom and Milgrom (1987).

[26] As discussed in Chapter 1, actual compensation contracts probably result from a range of competing pressures beyond effort incentivisation. Amongst other things, the contracts must be competitive in the "market for talent" and may be distorted by managerial power and the wider political climate.

Additionally, the flexibility of the exponential probability density function for output can be increased by introducing an extra parameter k. Incorporating k, the probability density function becomes:

$$f(q_i|k,a_i) = \left\{ \begin{array}{cc} \frac{1}{ka_i}e^{-\frac{q_i}{ka_i}}, & q_i \geq 0 \\ 0, & q_i < 0 \end{array} \right\}$$

[27]

The parameter k allows the marginal impact of effort on the distribution of output to vary. An increase in k increases both expected output and the variance of output. A numerical analysis similar to that in section 2.5 has been performed using this more general probability density function. More detail regarding the derivation of the equilibrium conditions when using $f(q_i|k,a_i)$ is provided in Appendix 2.6. Interestingly, the expected cost of inducing a given effort level is independent of k. This outcome appears to result from the optimal incentive contract, $w^*(q_i)$, changing with k to offset the impact of changes to the probability density function.

The key result is that as k is increased, the market size, B, at which moral hazard comes to have a greater downward impact on expected output than maximisation of joint profits, increases. The intuition behind this result is that although changing k has no impact on the agency costs to induce a given a_i, it does alter the pricing externality between firms. As k is increased, for a given effort level, a_i, the pricing externality grows. The pricing externality grows because for a given value of a_i expected output is increased. This result is illustrated in Figure 2.6 where the expected outputs when $k = 5$ and $k = 10$ are compared.

[27] The probability density function in section 2.3 can be obtained by setting $k = 1$.

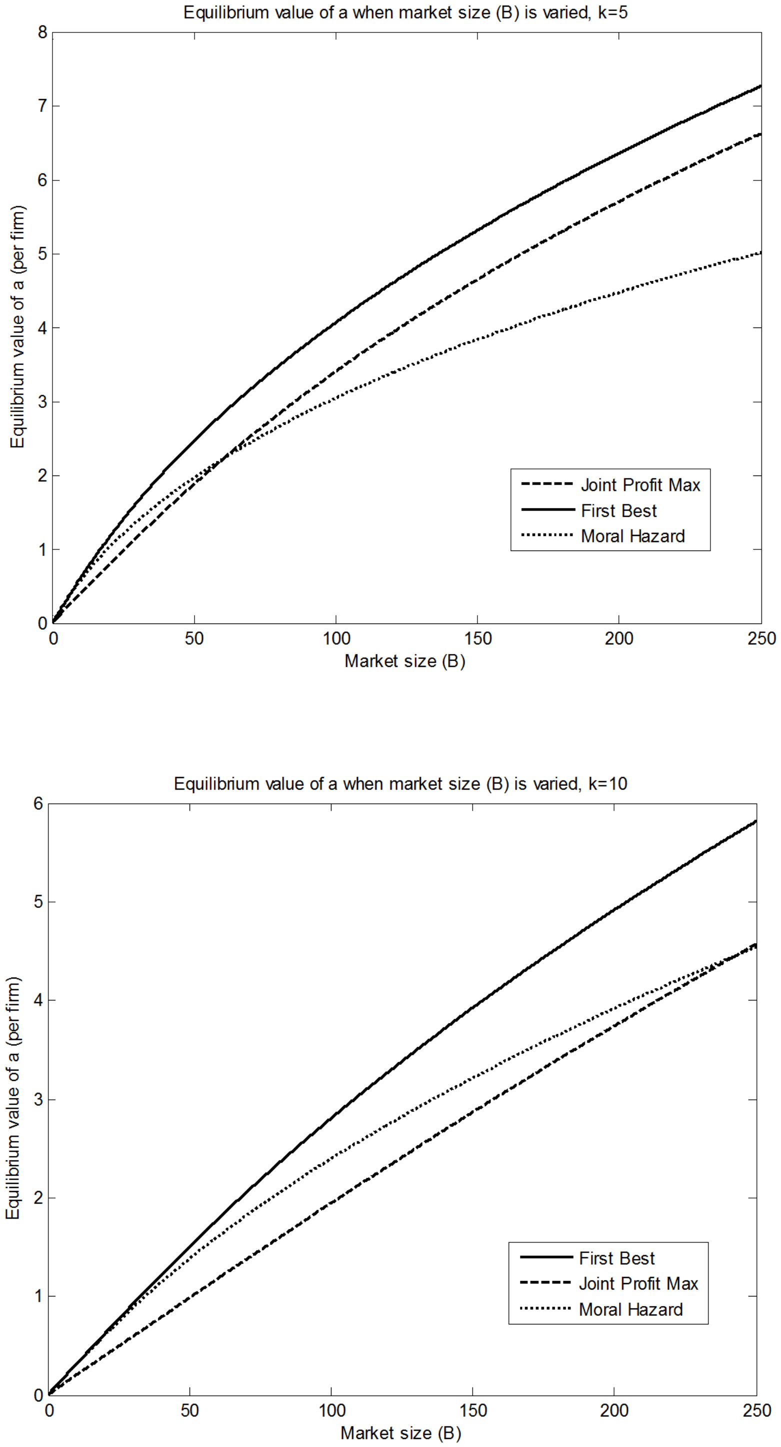

Figure 2.6: Equilibrium expected output per firm as market size, B, increases for k=5 and k=10.

Lastly, as in Chapter 1, the model also demonstrates the potential role of incentive contracts in product market collusion. In the current model, if a firm wanted to restrict output as part of a collusive arrangement, they would achieve this by reducing the strength of workers' incentives. In turn, this would reduce their agent's effort and the firm's expected output.

2.7. Conclusion

This chapter's novelty is to compare the impact on market outcomes of firms coordinating their output decisions, with the impact on market outcomes of firms facing internal agency problems. The central finding from the linear demand model is that at large market sizes moral hazard has a much greater downward impact on expected output than collusion between firms.

Having said this, the results from the exponential demand model suggest possible qualifications to this relationship between market size and the relative impact of moral hazard on market outcomes. Increasing the number of firms, or increasing the elasticity of price with respect to quantity, causes the impact of moral hazard on market outcomes to become relatively less important compared to collusion between firms. The chapter's key technical contribution, compared to other models combining moral hazard and competition, is to derive an equilibrium non-linear incentive contract when effort and output are continuous variables.

Importantly, the model goes on to highlight that firms' private incentives when deciding between monitoring and incentive contracts may not always be aligned with welfare maximisation. This is notable since until the Financial Crisis in 2008, concerns about managerial incentives generally focused on their potential suboptimality from the perspective of shareholders. The non-investment in monitoring displayed here, is an example of firms' incentive contracting decisions having an impact on consumer welfare. However, there is some distance between recognising that this externality exists, and suggesting that policymakers can take concrete actions to address any sub-optimality.

Policy intervention regarding collusion is relatively straightforward once collusion has been proven. A standardised policy response - collusion should be stopped - can be applied since product market collusion is unambiguously bad. Evaluating whether the welfare gain from a reduction in agency costs outweighs the investment costs required to achieve it, is a more challenging proposition, and one which is context specific. Nevertheless, it does seem important for policymakers and regulators to note that, in specific markets, private firms' solutions to agency problems may not be optimal from a welfare perspective.

Highlighting the significance of agency costs on product market outcomes also reaffirms the importance of moves to improve the oversight of managerial pay and performance. Increased reporting requirements and binding votes on pay should hopefully give greater priority to agency cost reductions.[28]

[28]Moves in this direction include increased disclosure rules for compensation in US proxy statements (see SEC Release No.33-8732A) and, in the UK, the introduction of binding "say on pay" votes (see "Cable plans binding votes on executive pay", Financial Times, 20 June 2012).

Overall, this chapter emphasises the importance of considering the internal workings of firms when addressing industrial organisation questions. The potentially significant impact of firms' internal agency costs on market outcomes seems an issue which has been underplayed in the past. The results presented suggest that more research is warranted into this reason why consumers, not just shareholders, have a legitimate interest in the way firms resolve agency issues.

2.8. Appendices

2.8.1. Appendix 2.1 - Generalised Proof of Existence Using Frank and Quandt (1963)

The model

Consider a generalised n-firm setting where agents and firms can be heterogeneous. The general model is identical to that of the parameterised model unless otherwise stated.

Given the n-firm setting, and in contrast to Section 2.3, let $Q = \sum_{k=1}^{n} q_k$ and $A = \sum_{k=1}^{n} a_k$. Continue to assume that demand is linear and continuous:

$$P(Q) = \left\{ \begin{array}{cc} B - Q, & B \geq Q \\ 0, & B < Q \end{array} \right\}$$

When $P(Q) = 0$ a finite quantity of the good is demanded. As a result, there is a positive real number $M < \infty$ such that $E\left[P(Q|A)\right] = 0$ for all $A \geq M$.

The probability density function for output given a specific effort level is $f(q_i|a_i)$. Assume that $f(q_i|a_i)$ is continuous and differentiable. Also, assume that the cumulative distribution function $F(q_i|a_i)$ satisfies $F_{a_i}(q_i|a_i) \leq 0$ for all q_i and that for some q_i $F_{a_i}(q_i|a_i) < 0$ holds. The support of the distribution is $[\underline{q}, \infty)$ where $\underline{q} \geq 0$ thus ruling out negative quantities.[29] The distribution's support remains constant regardless of effort.

[29]This assumption rules out, for example, the normal distribution.

The cost of effort function, $V_i(.)$, is now assumed to be smooth, continuous, increasing and convex. The first derivative of the agent's utility function with respect to wealth, $U_i'(w)$, must be invertible.

Solving the model

To solve the model follow Holmstrom (1979), but split the principal's problem into two steps. The first step involves the principal deriving the cost-minimising incentive contract, $w_i^*(q_i)$, to induce a given effort a_i. For principal i this problem can be stated as:

$$\max_{w_i(q_i)} \int_0^{\infty} -w_i(q_i)dF(q_i|a_i) \tag{2.10}$$

subject to PC:

$$\int_0^{\infty} U_i(w_i(q_i))dF(q_i|a_i) - V_i(a_i) \geq R_i \tag{2.11}$$

and ICC:

$$\int_0^{\infty} U_i(w_i(q_i))dF_{a_i}(q_i|a_i) - V_i'(a_i) = 0 \tag{2.12}$$

The second step is for the principal to select the optimal contract parameter (the optimal effort to induce) to maximise profits. This problem can be written as:

$$\max_{a_i} E(\pi_i) = \int_0^{B} ... \int_0^{B-Q_{-i-j}} \int_0^{B-Q_{-i}} (B-Q)\, q_i \Pi_{k=1}^{n} dF(q_k|a_k) \tag{2.13}$$

$$- \int_0^{\infty} w_i^*(q_i)dF(q_i|a_i)$$

where $Q = \sum_{k=1}^{n} q_k = q_1 + ... + q_i + q_j + ... + q_n$ and Q_{-i} is the same summation but excluding q_i.[30]

When solving the model the objective is to obtain a pure strategy Nash equilibrium in terms of the contract parameters selected by each principal.

Step 1: Deriving the cost-minimising contract

It is assumed that Jewitt's (1988) conditions for the validity of the first-order approach hold.

Lemma A2.1 *The cost-minimising incentive contract is:*

$$w_i^*(q_i) = U_i'^{-1}\left[\frac{1}{\lambda_i + \mu_i \frac{f_{a_i}(q_i|a_i)}{f(q_i|a_i)}}\right]$$

Given suitable assumptions on the agent's utility function and the distribution function, this optimal contract will be strictly convex. It is assumed these conditions are met.

Proof. Principal i's problem as described by (2.10)-(2.12) can be expressed as the following Lagrangian :

$$\max_{w(q_i)} L_i = \int_0^{\infty} -w(q_i)dF(q_i|a_i) + \lambda_i\left[\int_0^{\infty} U_i(w_i(q_i))dF(q_i|a_i) - V_i(a_i) - R_i\right] \tag{2.14}$$

$$+\mu_i\left[\int_0^{\infty} U_i(w_i(q_i))dF_{a_i}(q_i|a_i) - V_i'(a_i)\right]$$

[30] If an exponential inverse demand curve is used the expected revenue function becomes $E(R_i) = \int_0^{\infty} ... \int_0^{\infty} \left(Be^{-\varepsilon Q}\right) q_i \Pi_{k=1}^{n} dF(q_k|a_k)$. Otherwise the proof remains the same.

Holding a_i fixed, the necessary condition for maximisation is found by taking the partial derivative of (2.14) with respect to $w_i(q_i)$ and setting equal to zero:

$$\frac{\partial L_i}{\partial w(q_i)} = -\int_0^\infty dF(q_i|a_i) + \lambda_i \int U_i'(w_i(q_i))dF(q_i|a_i)$$

$$+\mu_i \int U_i'(w_i(q_i))dF_{a_i}(q_i|a_i) = 0$$

Realising that $dF(q_i|a_i) = f(q_i|a_i)dq_i$, dividing throughout by $U_i'(w_i(q_i))f(q_i|a_i)$ and re-arranging gives:

$$\frac{1}{U_i'(w_i(q_i))} = \lambda_i + \mu_i \frac{f_{a_i}(q_i|a_i)}{f(q_i|a_i)} \tag{2.15}$$

Re-arranging (2.15) to make $w_i(q_i)$ the subject, the cost-minimising contract is:

$$w_i^*(q_i) = U_i'^{-1}\left[\frac{1}{\lambda_i + \mu_i \frac{f_{a_i}(q_i|a_i)}{f(q_i|a_i)}}\right] \tag{2.16}$$

The values of λ_i and μ_i are found by inserting (2.16) into the ICC and PC and solving as two equations in two unknowns. Jewitt (1988) provides a proof for $\mu_i > 0$ which is included in Appendix 2.2. Given $\mu_i > 0$, the cost-minimising incentive contract is a strictly convex function of q_i if the agent's risk tolerance increases at a sufficiently fast rate and $\frac{f_a(q_i|a_i)}{f(q_i|a_i)}$ is a linear increasing function in q_i. The proof of this and a definition of risk tolerance, both from Basu et al (1985), are also provided in Appendix 2.2. ■

Step 2: Selecting the optimal value of a_i

Assuming that Basu et al's (1985) conditions for the strict convexity of $w_i^*(q_i)$ hold, Jewitt's conditions on the distribution function imply that the expected wage costs, $\int_0^\infty w_i^*(q_i)dF(q_i|a_i)$ in (2.13), will be convex in a_i. As such, if the expected revenue function,

$$E\left(R_i\right)=\int_0^B \ldots \int_0^{B-Q_{-i,-j}} \int_0^{B-Q_{-i}} \left(B-Q\right) q_i \Pi_{k=1}^n dF(q_k|a_k)$$

is strictly concave the principal's problem in (2.13) will be strictly concave in the contract parameter a_i. Technically, all that is required for equilibrium is for $E\left(\pi_i\right)$ to be strictly quasiconcave. Strict quasiconcavity ensures there is a unique profit-maximising value of a_i. Adding the assumption that $E\left(\pi_i\right)$ includes a stationary point, the FOC, $\frac{\partial E(\pi_i)}{\partial a_i}=0$, is a necessary and sufficient condition for profit maximisation,[31] and there is a unique profit-maximising value of a_i.

Demonstrating existence

This proof, adapted from Frank and Quandt (1963), involves demonstrating that the assumptions of the Kakutani fixed-point theorem hold for the current model.

Let the set $\Gamma_i=\{a_i\}$ of possible contract parameters be closed and connected for all a_i. Also define the following:

- a_i^* as the profit-maximising value of a_i for the ith firm if for some fixed $\phi_j = (a_1, ..., a_{i-1}, a_{i+1}, ..., a_n)$, $E\left[\pi_i\left(a_i^*, \phi_j\right)\right] \geq E\left[\pi_i\left(a_i, \phi_j\right)\right]$ for all a_i.

- If an a_i^* exists, and if $a_i^* \in g_i\left(\phi_j\right)$, then g_i is the ith firm's reaction correspondence.

- $\alpha=(a_1, ..., a_n)$ and $\alpha^*=(a_1^*, ..., a_n^*)$

- The mapping $G:\{\alpha\}\rightarrow\{\alpha^*\}$ is given by the mappings $g_1:\{\phi_1\}\rightarrow\{a_1^*\}, ..., g_i: \{\phi_i\}\rightarrow\{a_i^*\}, ..., g_n:\{\phi_n\}\rightarrow\{a_n^*\}$

[31] This also assumes $\frac{\partial E(\pi_i)}{\partial a_i}=0$ holds within the range of a_i considered.

An equilibrium exists in the principals' contract parameter game if the Kakutani Fixed Point Theorem applies to the mapping G and, hence, that the mapping G has a fixed point $G(\alpha) = \alpha$.

Lemma A2.2 *The sets* $\{a_i^*\}$ *are bounded for each* i.

Proof. By assumption, $a_i \in [\underline{a}, \overline{a}]$ and so the potential effort of each agent is bounded. Also, by assumption, each firm is required to employ an agent and must compensate them for their effort. As such, there is a lower bound to the contract parameter set at $a_i^* = \underline{a}$. Since a profit-maximising firm will always pay the agent just enough to maintain the agent's reservation utility, and no more, there is also an upper-bound to the contract parameter set at $a_i^* = \overline{a}$. ■

As such, it is sufficient to consider only the bounded sets of contract parameters $\Gamma_i' = \{a_i | \underline{a} \leq a_i \leq \overline{a}\}$. If any a_i^* exists, it must be that $a_i^* \subset \Gamma_i'$.

Lemma A2.3 *The sets* $\{a_i^*\}$ *contain at least one element.*

Proof. The functions $E\left[\pi_i\left(a_i, \phi_j\right)\right]$ are bounded by the assumptions that the expected profit function is continuous and strictly quasiconcave, and by Lemma A2.2. By the continuity of both the demand function and the expected cost of the incentive contract, $E\left[\pi_i\left(a_i, \phi_j\right)\right]$ must have a closed graph. As such, $E\left[\pi_i\left(a_i, \phi_j\right)\right]$ has a largest element and so there exists an a_i^* such that $E\left[\pi_i\left(a_i^*, \phi_j\right)\right] \geq E\left[\pi_i\left(a_i, \phi_j\right)\right]$.
■

Recall $\Gamma_i' = \{a_i\}$ is the bounded set of possible contract parameters. Define $\Gamma' = \{\alpha = (a_1, ..., a_n) | a_1 \in \Gamma_1', ..., a_n \in \Gamma_n'\}$.

Lemma A2.4 *The mapping $G : \Gamma' \to \Gamma'$ has a fixed point.*

Proof.

1. Γ' is closed by assumption and is bounded due to Lemma A2.2.

2. The mapping G maps the points $\alpha \in \Gamma'$ into sets of Γ'. This holds since $G(\alpha) \in \{\alpha^*\}$ by definition and $\{\alpha^*\} \subset \Gamma'$.

3. The set Γ' is convex. This holds because Γ' is the Cartesian product of intervals.

4. The image sets $G(\Gamma')$ are convex. Assume that there are two points $\alpha^* \in G(\alpha)$ and $\alpha^{**} \in G(\alpha)$. Then for each component

$$E\left[\pi_i\left(a_i^*, \phi_j\right)\right] \geq E\left[\pi_i\left(a_i, \phi_j\right)\right];$$

$$E\left[\pi_i\left(a_i^{**}, \phi_j\right)\right] \geq E\left[\pi_i\left(a_i, \phi_j\right)\right];$$

and choosing γ, $0 < \gamma < 1$,

$$E\left[\pi_i\left(a_i, \phi_j\right)\right] \leq \min\left\{E\left[\pi_i\left(a_i^*, \phi_j\right)\right], E\left[\pi_i\left(a_i^{**}, \phi_j\right)\right]\right\} < E\left[\pi_i\left(\gamma a_i^* + (1-\gamma) a_i^{**}, \phi_j\right)\right],$$

where the latter inequality holds due to the strict quasiconcavity of $E\left[\pi_i\left(a_i, \phi_j\right)\right]$. As a result, the entire line segment between a_i^* and a_i^{**} is in the image $G(\alpha)$.

5. G is upper semi-continuous. For this to be true it is necessary that for every sequence $\alpha^\rho \to \alpha^0$, such that $\alpha^{*\rho} \in G(\alpha^\rho)$ and $\alpha^{*\rho} \to \alpha^{*0}$, it is the case that $\alpha^{*0} \in G(\alpha^0)$. Follow a proof by contradiction. If the contrary held then

$$E\left[\pi_i\left(a_i^{*0},\phi_j^0\right)\right] < E\left[\pi_i\left(g_i\left(\phi_j^0\right),\phi_j^0\right)\right]$$

for some i. As a result, there would be points $\alpha^{*\rho}$ arbitrarily close to α^{*0} for which $\alpha^{*\rho} \notin G(\alpha^\rho)$, which is a contradiction.

So, by the Kakutani Fixed Point Theorem, there exists α such that $G(\alpha) = \alpha$. As such, the mapping G has a fixed point. ■

Existence of equilibrium in a generalised n-firm setting with linear demand has been demonstrated under suitable assumptions.

2.8.2. Appendix 2.2 - Proofs by Other Authors

For ease of notation the i subscript is dropped in this Appendix.

Conditions for the Validity of the First-Order Approach - Jewitt (1988)

Jewitt states that the conditions for the first-order approach to be valid are:

(i) $\int_{-\infty}^{q} F(q|a)dq$ is non-increasing convex in a for each value of q,

(ii) $\int qdF(q|a)dq$ is non-decreasing concave in a,

(iii) $\frac{f_a(q|a)}{f(q|a)}$ is non-decreasing concave in q for each value of a, and

(iv) the utility of the agent is a concave increasing function of the observable variables, i.e. $\omega(z) = U\left(U'^{-1}\left(\frac{1}{z}\right)\right)$, where $z > 0$, is concave.

These conditions essentially ensure the agent's problem is concave. To understand how they do this, let $w(q)$ solve the first-order condition:

$$\int_0^\infty U\left(w\left(q\right)\right) dF_a\left(q|a\right) - V'\left(a\right) = 0 \tag{2.17}$$

Using the fact that $\mu > 0$ [32], condition (iii) and that for the cost-minimising contract:

$$\frac{1}{U'\left(w\left(q\right)\right)} = \lambda + \mu \frac{f_a\left(q|a\right)}{f\left(q|a\right)} \tag{2.18}$$

[33] it is possible to say that $\frac{1}{U'(w(q))}$ is non-decreasing concave in q. Condition (iv) is the requirement that $U\left(w(q)\right)$ is a concave transformation of $\frac{1}{U'(w(q))}$. Since the class of non-decreasing concave functions is closed under composition, $U\left(w(q)\right)$ is non-decreasing concave in q. Lastly, it is necessary to demonstrate that the transformation φ to φ^* defined by:

$$\varphi^*(a) = \int_0^\infty \varphi(q) dF(q, a)$$

preserves concavity. If it does, the agent's problem is concave and the solution to the FOC of the agent's utility maximisation problem is a global maximum. Conditions (i) and (ii) are necessary and sufficient for this concavity-preserving property to hold.

■

[32] See the following sub-section of this appendix for Jewitt's proof that $\mu > 0$.
[33] For the derivation of this condition see the proof of Lemma A2.1.

Proof that $\mu > 0$ - Jewitt (1988)

Lemma A2.5 *If U is an increasing concave function and $V'(a) > 0$, then any μ satisfying*

$$\int_0^\infty U\left(w\left(q\right)\right) dF_a\left(q|a\right) - V'\left(a\right) = 0$$

and

$$\frac{1}{U'\left(w\left(q\right)\right)} = \lambda + \mu \frac{f_a\left(q|a\right)}{f\left(q|a\right)}$$

is positive.

Proof. Re-arranging (2.18) gives:

$$f_a(q|a) = \frac{1}{\mu}\left(\frac{1}{U'(w(q))} - \lambda\right) f(q|a)$$

Substituting this into (2.17) gives:

$$\int U(w(q))\left(\frac{1}{U'(w(q))} - \lambda\right) f(q|a)dq = \mu V'(a) \tag{2.19}$$

Since $\int f(q|a)dq = 1$ it must be the case that $\int f_a(q|a)dq = 0$ and in turn:

$$E\left(\frac{f_a(q|a)}{f(q|a)}\right) = 0$$

So considering (2.18) it is possible to write:

$$E\left(\frac{1}{U'(w(q))}\right) = \lambda \tag{2.20}$$

Now consider the LHS of (2.19):

$$E\left[\int U(w(q))\left(\frac{1}{U'(w(q))}-\lambda\right)f(q|a)dq\right]$$

$$=\int \frac{U(w(q))}{U'(w(q))}f(q|a)dq-\lambda\int U(w(q))f(q|a)dq$$

Given (2.20), this expression gives the covariance between $U(w(q))$ and $\frac{1}{U'(w(q))}$. As a result:

$$Cov\left(U(w(q)),\frac{1}{U'(w(q))}\right)=\mu V'(a)$$

Since both $U\left(.\right)$ and $\frac{1}{U'(.)}$ are monotone increasing functions, they must have a non-negative covariance and, since $V'(a)>0$ by assumption, it follows that $\mu\geq 0$. $\mu=0$ can be ruled out since it would imply that $w(q)$ would be a constant by re-arrangement of (2.18). Having $w\left(q\right)$ as a constant would violate the incentive compatibility constraint and, hence, $\mu>0$ must hold. ■

Conditions for the Cost-Minimising Incentive Contract to be Convex - Basu et al (1985)

The following lemma is taken, with changed notation, from Appendix A of Basu et al (1985).

Lemma A2.6 *If* $\frac{f_a(q|a)}{f(q|a)}$ *is a linear function of* q *over some interval, then the cost-minimising contract,* $w(q)$*, is a strictly convex function of* q *over that interval, if the rate of change of risk tolerance,* $T'(w)$*, exceeds one.*

Proof. Assume $\frac{f_a(q|a)}{f(q|a)}$ is linear in q. This means (2.18) can be written as:

$$\frac{1}{U'(w(q))}=A+Bq$$

Taking the second derivative with respect to q of both sides of this expression gives:

$$\frac{U'''(w)\left[w'(q)\right]^2 + U''(w)w''(q)}{\left[U'(w)\right]^2} - \frac{2\left[U''(w)\right]^2\left[w'(q)\right]^2}{\left[U'(w)\right]^3} = 0$$

Dividing throughout by $\frac{U''(w)}{[U'(w)]^3}$ and simplifying leads to:

$$U'(w)w''(q) = \left[2U''(w) - \frac{U'(w)U'''(w)}{U''(w)}\right]\left[w'(q)\right]^2$$

Since $U'(w) > 0$ and $U''(w) < 0$:

$$sign\left[w''(q)\right] = sign\left[\frac{U'(w)U'''(w)}{\left[U''(w)\right]^2} - 2\right] \tag{2.21}$$

The Arrow-Pratt measure of absolute risk aversion is given by:

$$R_a(w) = \frac{-U''(w)}{U'(w)}$$

Since $R_a(w)$ measures risk aversion, its inverse, $T(w)$, given by:

$$T(w) = \frac{1}{R_a(w)} = \frac{-U'(w)}{U''(w)},$$

measures risk tolerance. Differentiating $T(w)$ with respect to w gives:

$$T'(w) = \frac{U'(w)U'''(w)}{\left[U''(w)\right]^2} - 1$$

Combining this with (2.21) leads to:

$$sign\left[w''(q)\right] = sign\left[T'(w) - 1\right]$$

and so it follows that $w''(q) > 0$ if, and only if, $T'(w) > 1$. ■

A power utility function of the form $U = \frac{1}{\delta} w^{\delta}$, where $0 < \delta < 1$, displays the necessary risk tolerance.

2.8.3. Appendix 2.3 - Proof of FOCs in the Parameterised Model

The First Best (Observable and Verifiable Effort)

Lemma A2.7 *When effort is observable and verifiable the condition for a symmetric equilibrium is:*

$$\frac{1}{a^2} e^{-\frac{B}{a}} \left(\frac{1}{3} B^3 + \frac{3}{2} B^2 a + 4Ba^2 + 5a^3 \right) + B - 5a - a^3 = 0$$

Proof. Solve the principal's problem in two steps.

Step 1: Find the cost-minimising contract to induce a given effort a_i from agent i. Since effort is observable and verifiable, principal i can induce agent i to exert an effort a_i by offering a forcing contract. This contract involves a payment just satisfying agent i's PC if the effort a_i is exerted, and a payment of zero if any other effort level is observed. There is no need to write a contract linked to output. Denote w^* the fixed payment paid when $a_i = a_i^*$ is observed. For the agent's PC to be satisfied with equality requires:

$$2\left(w_i^*\right)^{\frac{1}{2}} - a_i^2 - R = 0$$

This gives the optimal fixed payment as:

$$w_i^* = \left(\frac{a_i^2 + R}{2}\right)^2$$

The full forcing contract is:

$$w_i^*(a_i) = \left\{ \begin{array}{cc} \left(\frac{a_i^2+R}{2}\right)^2 & \text{if } a_i = a_i^* \\ 0 & \text{otherwise} \end{array} \right\} \tag{2.22}$$

Step 2: Now principal i's problem is to select the optimal value of a_i to set as a_i^*. The expected revenue function, $E(R_i)$, is unchanged from section 2.3. Setting $R = 0$, the expected cost of inducing an effort a_i is simply:

$$\left(\frac{a_i^2}{2}\right)^2$$

Principal i's second step problem is, therefore:

$$\max_{a_i} E(\pi_i) = a_i a_j^3 \frac{e^{-\frac{B}{a_j}}}{(a_i - a_j)^2} \tag{2.23}$$

$$+ a_i^2 e^{-\frac{B}{a_i}} \frac{Ba_i - Ba_j + 2a_i^2 - 3a_i a_j}{(a_i - a_j)^2} + (B - 2a_i - a_j) a_i - \left(\frac{a_i^2}{2}\right)^2$$

The resulting FOC is:

$$\frac{\partial E(\pi_i)}{\partial a_i} = -a_j^3 \frac{e^{-\frac{B}{a_j}}}{(a_i - a_j)^3}(a_i + a_j)$$

$$+ \frac{e^{-\frac{B}{a_i}}}{(a_i - a_j)^3} \left(\begin{array}{c} B^2 a_i^2 - 2B_i^2 a_i a_j + B^2 a_j^2 + 3Ba_i^3 \\ -8Ba_i^2 a_j + 5Ba_i a_j^2 + 4a_i^4 - 11a_i^3 a_j + 9a_i^2 a_j^2 \end{array} \right)$$

$$+B - 4a_i - a_j - a_i^3 = 0$$

Appealing to the problem's symmetry, combining the first two terms, and applying l'Hôpital's rule three times, the condition for a symmetric equilibrium reduces to:

$$\frac{1}{a^2}e^{-\frac{B}{a}}\left(\frac{1}{3}B^3 + \frac{3}{2}B^2a + 4Ba^2 + 5a^3\right) + B - 5a - a^3 = 0$$

■

Firms Maximise Joint Profits (Collusion)

Lemma A2.8 *When firms maximise joint profits, and effort is observable and verifiable, the condition for a symmetric equilibrium is:*

$$\frac{1}{2a^2}e^{-\frac{B}{a}}\left(B^3 + 4B^2a + 10Ba^2 + 12a^3\right) + B - 6a - a^3 = 0$$

Proof. **Step 1:** Since effort is observable and verifiable, the cost-minimising contracts are again forcing contracts of the form in (2.22):

$$w_i^*(q_i) = \left(\frac{a_i^2}{2}\right)^2 \text{ and } w_j^*(q_j) = \left(\frac{a_j^2}{2}\right)^2$$

Step 2: To find the optimal values for a_i and a_j assume the firms act as a single monopolist with two agents. The problem facing the combined firm is:

$$\max_{a_i, a_j} E\left(\pi_{i+j}\right) = E\left(R_i\right) + E\left(R_j\right) - \left(\frac{a_i^2}{2}\right)^2 - \left(\frac{a_j^2}{2}\right)^2$$

where

$$E(R_i) + E(R_j) = \frac{1}{a_i - a_j}\left(a_i^2 e^{-\frac{B}{a_i}}(B + 2a_i) - a_j^2 e^{-\frac{B}{a_j}}(B + 2a_j)\right)$$

$$+a_i(B - 2a_i - a_j) + a_j(B - a_i - 2a_j)$$

There are two FOCs for this problem, $\frac{\partial E(\pi_{i+j})}{\partial a_i} = 0$ and $\frac{\partial E(\pi_{i+j})}{\partial a_j} = 0$. The FOC with respect to a_i is:

$$\frac{\partial E(\pi_{i+j})}{\partial a_i} = \frac{1}{(a_i - a_j)^2}\left(\begin{array}{c} e^{-\frac{B}{a_i}}(4a_i^3 - 6a_i^2 a_j + 3Ba_i^2 + B^2 a_i - B^2 a_j - 4Ba_i a_j) \\ -a_j^2 e^{-\frac{B}{a_j}}(B + 2a_j) \end{array}\right)$$

$$+B - 4a_i - 2a_j - a_i^3 = 0$$

Applying symmetry and using l'Hôpital's rule three times, the FOC for maximisation of joint profits becomes:

$$\frac{1}{2a^2}e^{-\frac{B}{a}}\left(B^3 + 4B^2 a + 10Ba^2 + 12a^3\right) + B - 6a - a^3 = 0$$

■

2.8.4. Appendix 2.4 - Proof of Result 2.2

Result A2.1 *The ranges of investment costs where a social planner would invest in a perfect monitoring technology, but two competing firms would not, are:*

Market Size (B)	Investment Cost Range
50	$37.5 < C < 45.6$
100	$113.9 < C < 125.6$
200	$308.8 < C < 333.9$

Proof. The proof is split into two parts. The first involves finding the ranges of investment costs where different Nash equilibria occur in the investment subgame. The second is to find the ranges of investment costs where a social planner maximising total surplus would choose to invest/not invest. Comparing these cost ranges then leads to Result A2.1. The proof described is for $B = 100$. For other values of B the procedure is qualitatively identical.[34]

Lemma A2.9 *In the initial investment subgame:*

- *if* $C < 111.0$ *Invest is the strictly dominant strategy and the Nash equilibrium is:* $(Invest, Invest)$
- *if* $111.0 < C < 113.9$ *there are two Nash equilibria:* $(Invest, NotInvest)$ *and* $(NotInvest, Invest)$
- *if* $C > 113.9$ *NotInvest is the strictly dominant strategy and the Nash equilibrium is:* $(NotInvest, NotInvest)$

Proof. To determine the equilibrium decisions of each firm, the pay-off matrix for the two principals in the investment subgame must be formed. This involves identifying the optimal contract parameters, a_i^* and a_j^*, which maximise the expected

[34] The full workings for $B = 50$ and $B = 200$ are available on request.

profits of each firm given different combinations of investment decisions. If firm i chooses $NotInvest$, a_i is selected to maximise (2.7) and, if firm i chooses $Invest$, a_i is selected to maximise (2.23).[35] The values of a_i^* and a_j^* for each of the decision pairs, when $B = 100$, are shown below:

Decision Pairs	a_i^* , a_j^* to 3d.p.
$(Invest, Invest)$	$4.283, 4.283$
$(Invest, NotInvest)$	$4.312, 2.575$
$(NotInvest, Invest)$	$2.575, 4.312$
$(NotInvest, NotInvest)$	$2.592, 2.592$

Inserting a_i^* and a_j^* back into the firms' expected profit functions allows the firms' expected profits gross of investment costs to be obtained. Subtracting the investment cost, C, when a firm chooses $Invest$, gives the following pay-off matrix for the investment subgame:

		Principal j	
		$Invest$	$NotInvest$
Principal i	$Invest$	$289.1 - C, 289.1 - C$	$296.5 - C, 178.2$
	$NotInvest$	$178.2, 296.5 - C$	$182.6, 182.6$

By comparing the pay-offs between the different decision pairs, the Nash equilibria of the investment subgame can be characterised for different values of C. This gives Lemma A2.9. ■

Now consider the decision of a social planner maximising total surplus. The social planner has three options: invest in neither firm, invest in one firm, or invest in both

[35]When only one firm invests, the problem is no longer symmetric and so $a_i^* = a_j^*$ no longer holds. The two first-order conditions with respect to a_i and a_j are therefore solved as a system of two equations in two unknowns.

firms. For now, assume that the social planner can only make different investment decisions compared to profit-maximising firms.[36] As a result the values of a_i^* and a_j^*, stated in the proof of Lemma A2.9, are still used in the calculations below.

Lemma A2.10 *A social planner maximising total surplus will:*

- *invest in the monitoring technology for both firms if* $C < 122.3$
- *invest in the monitoring technology for one firm if* $122.3 < C < 125.6$
- *not invest if* $C > 125.6$

Total surplus gross of investment costs is given by:

$$E\left(TS_g\right) = \int_0^B \int_0^{B-q_j} \left(B\left(q_i+q_j\right) - \frac{\left(q_i+q_j\right)^2}{2}\right) \frac{1}{a_i} e^{-\frac{q_i}{a_i}} \frac{1}{a_j} e^{-\frac{q_j}{a_j}} dq_i dq_j$$

$$+\frac{(B)^2}{2}\left(\frac{1}{a_i-a_j}\left(a_i e^{-\frac{B}{a_i}} - a_j e^{-\frac{B}{a_j}}\right)\right) - E\left(w_i\left(q_i\right)\right) - E\left(w_j\left(q_j\right)\right)$$

where

$$B\left(q_i+q_j\right) - \frac{\left(q_i+q_j\right)^2}{2}$$

is the area under the inverse demand curve, when $q_i + q_j \leq B$ and

$$\frac{(B)^2}{2}\left(\frac{1}{a_i-a_j}\left(a_i e^{-\frac{B}{a_i}} - a_j e^{-\frac{B}{a_j}}\right)\right)$$

is the area under the inverse demand curve, when $q_i + q_j > B$, multiplied by the probability of $q_i + q_j > B$. The probability that $q_i + q_j > B$ is:

[36] This rules out the possibility of the social planner directly setting effort/output levels. From a policy perspective, this seems a reasonable distinction to make. In the US and EU investment subsidies are fairly common, whereas the micro-managing of firms' operational decisions by policymakers is rare.

$$P(q_i + q_j > B) = 1 - P(q_i + q_j \leq B)$$

where

$$P(q_i + q_j \leq B) = \int_0^B \int_0^{B-q_j} \frac{1}{a_i} e^{-\frac{q_i}{a_i}} \frac{1}{a_j} e^{-\frac{q_j}{a_j}} dq_i dq_j = 1 - \frac{1}{a_i - a_j} \left(a_i e^{-\frac{B}{a_i}} - a_j e^{-\frac{B}{a_j}} \right)$$

After completing the necessary integration and substituting in the relevant values of a_i^* and a_j^*, the following values for total surplus net of investment costs are obtained:

Invest in both firms	Invest in one firm	Don't Invest
$633.3 - 2C$	$511.0 - C$	385.4

By comparing these values Lemma A2.9 is obtained. ■

Comparing the cost ranges between Lemmas A2.9 and A2.10 gives Result A2.1.

■

2.8.5. Appendix 2.5 - Proofs: Exponential Inverse Demand Curve

$E(\pi_i)$ is Strictly Quasiconcave

To demonstrate the expected profit function, $E(\pi_i)$, is strictly quasiconcave, firstly, derive $E(\pi_i)$ for the n-firm case.

Lemma A2.11 *The expected revenue function for firm i in an n-firm setting is:*

$$E(R_i) = \frac{Da_i}{(\tau a_i + 1)^2 \Pi_{k \neq i}^n (\tau a_k + 1)}$$

Proof. The expected revenue function for firm i in an n-firm setting can be written as:

$$E\left(R_i\right)=\int_0^{\infty}...\int_0^{\infty}\int_0^{\infty}De^{-\tau Q}q_i\Pi_{k=1}^n\frac{1}{a_k}e^{-\frac{qk}{a_k}}dq_k$$

where firm i is just a particular firm between 1 and n. Re-write this expression as:

$$E\left(R_i\right)=\int_0^{\infty}...\int_0^{\infty}\int_0^{\infty}De^{-\tau(q_i+Q_{-i})}q_i\frac{1}{a_i}e^{-\frac{q_i}{a_i}}dq_i\Pi_{k\neq i}^n\frac{1}{a_k}e^{-\frac{qk}{a_k}}dq_k$$

where $Q_{-i}=\left(\sum_{k=1}^n q_k\right)-q_i$. Now integrate with respect to dq_i. Since the probability density functions for the quantities of firms other than i do not involve q_i, they can be moved outside of the integral with respect to dq_i. These other integrals are dealt with subsequently. As such, the first integral to consider is:

$$\int_0^{\infty}De^{-\tau(q_i+Q_{-i})}q_i\frac{1}{a_i}e^{-\frac{q_i}{a_i}}dq_i=\int_0^{\infty}\frac{D}{a_i}e^{\frac{-(\tau a_i+1)q_i-\tau a_i Q_{-i}}{a_i}}q_idq_i=\frac{Da_i}{\left(\tau a_i+1\right)^2}e^{-\tau Q_{-i}}$$

After this first integration $E\left(R_i\right)$ can be written as:

$$E\left(R_i\right)=\int_0^{\infty}...\int_0^{\infty}\frac{Da_i}{\left(\tau a_i+1\right)^2}e^{-\tau(q_j+Q_{-i-j})}\frac{1}{a_j}e^{-\frac{q_j}{a_j}}dq_j\Pi_{k\neq i,j}^n\frac{1}{a_k}e^{-\frac{qk}{a_k}}dq_k$$

where $Q_{-i-j}=\left(\sum_{k=1}^n q_k\right)-q_i-q_j$. Applying the same separating procedure used for the integral with respect dq_i the integral with respect to dq_j gives:

$$\int_0^{\infty}\frac{Da_i}{\left(\tau a_i+1\right)^2}e^{-\tau(q_j+Q_{-i-j})}\frac{1}{a_j}e^{-\frac{q_j}{a_j}}dq_j \tag{2.24}$$

$$=\frac{Da_i}{\left(\tau a_i+1\right)^2}\int_0^{\infty}\frac{1}{a_j}e^{-\left(\frac{\left(\tau a_j+1\right)q_j+\tau a_j Q_{-i-j}}{a_j}\right)}dq_j$$

$$= \frac{Da_i}{(\tau a_i + 1)^2 (\tau a_j + 1)} e^{-\tau Q_{-i-j}}$$

and, hence, $E(R_i)$ becomes:

$$E(R_i) = \int_0^{\infty} ... \int_0^{\infty} \frac{Da_i}{(\tau a_i + 1)^2 (\tau a_j + 1)} e^{-\tau Q_{-i-j}} \Pi_{k \neq i,j}^n \frac{1}{a_k} e^{-\frac{q_k}{a_k}} dq_k$$

Integrating with respect to each subsequent dq_k is qualitatively identical to the operation performed when integrating with respect to dq_j. For example, if the next integration is with respect to dq_l the resulting expression, equivalent to (2.24), is:

$$\frac{Da_i}{(\tau a_i + 1)^2 (\tau a_j + 1)(\tau a_l + 1)} e^{-\tau Q_{-i-j-l}}$$

Once all the integration procedures have been performed, the expected revenue function can be expressed as:

$$E(R_i) = \frac{Da_i}{(\tau a_i + 1)^2 \Pi_{k \neq i}^n (\tau a_k + 1)}$$

■

Having found $E(R_i)$, and noting the principal-agent problem remains identical to that in section 2.3, the expected profit function for firm i when moral hazard is present is:

$$E(\pi_i) = \frac{Da_i}{(\tau a_i + 1)^2 \Pi_{k \neq i}^n (\tau a_k + 1)} - \frac{5}{4} a_i^4$$

By inspection this function is continuous. To demonstrate that this function is strictly quasiconcave in a_i, it is necessary to show that: (i) $\frac{\partial E(\pi_i)}{\partial a_i}$ changes sign only once over the range $a_i \in [\underline{a}, \overline{a}]$; (ii) this sign change is from positive to negative; and (iii) if $\frac{\partial E(\pi_i)}{\partial a_i} = 0$ holds, it holds at only one point.

The expression for $\frac{\partial E(\pi_i)}{\partial a_i}$ is:

$$\frac{\partial E\left(\pi_i\right)}{\partial a_i} = \frac{D\left(1-\tau a_i\right)}{\left(\tau a_i+1\right)^3 \Pi_{k\neq i}^{n}\left(\tau a_k+1\right)} - 5a_i^3$$

As long as $\underline{a} > 0$ is sufficiently small, $\frac{\partial E(\pi_i)}{\partial a_i}$ will start as a positive value.[37] By inspection, both $\frac{D(1-\tau a_i)}{(\tau a_i+1)^3 \Pi_{k\neq i}^{n}(\tau a_k+1)}$ and $-5a_i^3$ are strictly decreasing in a_i and so $\frac{\partial E(\pi_i)}{\partial a_i}$ is a strictly decreasing function in a_i. Assuming $\overline{a}$ is sufficiently large not to act as a constraint, then, as a_i grows large, $\frac{\partial E(\pi_i)}{\partial a_i}$ will turn negative and stay negative. Since $\frac{\partial E(\pi_i)}{\partial a_i}$ is a strictly decreasing function, this function will cross the horizontal axis, i.e. $\frac{\partial E(\pi_i)}{\partial a_i} = 0$, only once. Hence, the expected profit function is strictly quasiconcave over the range $a_i \in [\underline{a}, \overline{a}]$.

Since $E\left(\pi_i\right)$ is strictly quasiconcave, then, by Theorem 2.1, an equilibrium will exist in the principals' contract parameter choice game.

Uniqueness of the Two-Firm Equilibrium

For the two-firm case, the FOC for firm i to be profit-maximising is:

$$\frac{\partial E\left(\pi_i\right)}{\partial a_i} = \frac{D\left(1-\tau a_i\right)}{\left(\tau a_i+1\right)^3 \left(\tau a_j+1\right)} - 5a_i^3 = 0 \tag{2.25}$$

37 $\frac{\partial E(\pi_i)}{\partial a_i}$ will be positive whenever:

$$D > \frac{5a_i^3\left(\tau a_i+1\right)^3 \Pi_{k\neq i}^{n}\left(\tau a_k+1\right)}{\left(1-\tau a_i\right)}$$

As a_i tends to zero, the RHS of this inequality tends to zero. Hence, the requirement for $\frac{\partial E(\pi_i)}{\partial a_i}$ to be positive at $\underline{a}$ tends to the condition $D > 0$ as $\underline{a}$ becomes small.

Re-arranging this condition, a_j can be written as an explicit function of a_i:[38]

$$a_j = \frac{D\left(1-\tau a_i\right)}{5\tau a_i^3\left(\tau a_i+1\right)^3} - \frac{1}{\tau} \tag{2.26}$$

By symmetry, the equivalent equation for a_i, in terms of a_j, is:

$$a_i = \frac{D\left(1-\tau a_j\right)}{5\tau a_j^3\left(\tau a_j+1\right)^3} - \frac{1}{\tau} \tag{2.27}$$

To demonstrate a unique equilibrium exists, use a basic geometric argument. The aim is to prove that the lines described by (2.26) and (2.27) cross once, and only once, in (a_i, a_j)-space. Firstly, note that both equations are continuous. Also, note that because $a_i, a_j \in [\underline{a}, \overline{a}]$, both (2.26) and (2.27) are bounded and closed. Lastly, recall the starting assumption that $\underline{a}$ and $\overline{a}$ are sufficiently far apart never to impinge on the equilibrium outcome. Combining continuity, boundness, closedness and the symmetry of the problem, the lines (2.26) and (2.27) must cross at least once.

To demonstrate that the lines (2.26) and (2.27) cross only once involves demonstrating two things: (i) (2.26) and (2.27) are decreasing convex over the range $[\underline{a}, \overline{a}]$, and (ii) (2.26) and (2.27) do not coincide.

Lemma A2.12 *The functions (2.26) and (2.27) are both decreasing convex over the range* $a_i, a_j \in [\underline{a}, \overline{a}]$.

Proof. Due to symmetry, if one of (2.26) and (2.27) is proven to be decreasing convex, the other function will also be decreasing convex. Consider (2.26) only. The

[38]Note that (2.26) and (2.27) are not the reaction functions of principals i and j.

function is decreasing as long as:

$$\frac{\partial a_j}{\partial a_i} = -\frac{1}{5\tau}\frac{D}{a_i^4\left(\tau a_i + 1\right)^4}\left(-5\tau^2 a_i^2 + 4\tau a_i + 3\right) < 0$$

which is true if

$$-5\tau^2 a_i^2 + 4\tau a_i + 3 > 0 \tag{2.28}$$

By inspection of (2.26), a_j is guaranteed to be negative once $a_i \geq \frac{1}{\tau}$. Setting the LHS of (2.28) equal to zero and finding the roots of the resulting quadratic, one can say that when $\frac{4-2\sqrt{19}}{10\tau} < a_i < \frac{4+2\sqrt{19}}{10\tau}$ it implies $\frac{\partial a_j}{\partial a_i} < 0$. Since $\frac{4-2\sqrt{19}}{10\tau} < 0$ and $\underline{a} > 0$, we only need to check that $a_i < \frac{4+2\sqrt{19}}{10\tau}$ holds. Consider $a_i = \frac{1}{\tau}$. Since $\frac{1}{\tau} < \frac{4+2\sqrt{19}}{10\tau}$, it means that for any positive value of a_i, such that $a_j \geq \underline{a} > 0$, $\frac{\partial a_j}{\partial a_i} < 0$ holds. Hence, (2.26) is decreasing in a_i and (2.27) is decreasing in a_j, for the region of (a_i, a_j)-space being considered.

For (2.26) to be convex requires:

$$\frac{\partial^2 a_j}{\partial a_i^2} = \frac{6}{5\tau}\frac{D}{a_i^5\left(\tau a_i + 1\right)^5}\left(-5\tau^3 a_i^3 + 3\tau^2 a_i^2 + 6\tau a_i + 2\right) > 0$$

which holds if:

$$-5\tau^3 a_i^3 + 3\tau^2 a_i^2 + 6\tau a_i + 2 > 0 \tag{2.29}$$

Again, the restriction $a_i \geq \underline{a} > 0$ means it is sufficient to demonstrate (2.26) is convex when a_i satisfies $\underline{a} \leq a_i \leq \frac{1}{\tau}$. The argument for $\frac{\partial^2 a_j}{\partial a_i^2} > 0$ is that (2.29) is a cubic and so has two stationary points. The two stationary points of (2.29) are at $\frac{6-2\sqrt{99}}{30\tau}$

and $\frac{6+2\sqrt{99}}{30\tau}$. Only the second stationary point occurs at a positive value of a_i. By inspection, the a_i^3 term has a negative coefficient and, hence, when $|a_i|$ is large, (2.29) is decreasing. Hence, when $a_i \geq \underline{a}$, (2.29) is quasiconcave. Also note that when $a_i = \underline{a}$, for $\underline{a}$ small enough, the value of (2.29) is strictly greater than zero. When $a_i = \frac{1}{\tau}$, the value of (2.29) is also strictly greater than zero. Since (2.29) is strictly positive at $a_i = \underline{a}$ and at $a_i = \frac{1}{\tau}$, and is also quasiconcave between these two points, it means that for the relevant range of a_i, (2.29) is positive. This means that $\frac{\partial^2 a_j}{\partial a_i^2} > 0$ for the relevant region of (a_i, a_j)-space and (2.26) is decreasing convex as required. ■

As (2.26) and (2.27) are both decreasing convex it implies that they either coincide or cross only once.

Lemma A2.13 *The lines (2.26) and (2.27) do not coincide.*

Proof. Follow a proof by contradiction. Firstly, consider the case where $a_i \neq a_j$. Assume the lines do coincide. Then it must be the case that when either $a_i = \underline{a}$ or $a_j = \underline{a}$ both (2.26) and (2.27) must hold.

Consider the case where $a_j = \underline{a}$ and denote the value of a_i which solves (2.26) and (2.27) as $\widehat{a}_i$. Thus, (2.26) and (2.27) can be re-written as:

$$5\tau\widehat{a}_i^3\left(\tau\widehat{a}_i + 1\right)^3\left(\tau\underline{a} + 1\right) = D\tau\left(1 - \tau\widehat{a}_i\right) \tag{2.30}$$

and

$$5\tau\underline{a}^3\left(\tau\underline{a}+1\right)^3\left(\tau\widehat{a}_i+1\right)=D\tau\left(1-\tau\underline{a}\right) \tag{2.31}$$

Assume $\widehat{a}_i > \underline{a}$; this implies that the RHS of (2.30) has a smaller value than the RHS of (2.31). However, $\widehat{a}_i > \underline{a}$ also implies that the LHS of (2.30) has a higher value than the LHS of (2.31). As a result, there cannot be a value of $\widehat{a}_i$ which satisfies both (2.30) and (2.31). Hence, there is a contradiction: (2.26) and (2.27) do not coincide when $a_i > \underline{a}$. As $a_j = \underline{a}$, the case of $a_i < a_j$ does not need to be considered. If $a_j > \underline{a}$, making $a_i < a_j$ would not alter the logic of the proof. Also, by symmetry, the same arguments hold when we hold a_i fixed and vary a_j.

Now consider the case where $a_i = a_j = a$ and again assume both lines coincide. The only decreasing line that can satisfy these two conditions is a straight line decreasing at an angle of 45 degrees. The second derivative of such a line must be zero. However, from the proof of Lemma A2.12 it is known that for plausible values of $a_i, a_j > \underline{a}$: $\frac{\partial^2 a_j}{\partial a_i^2} > 0$ and $\frac{\partial^2 a_i}{\partial a_j^2} > 0$ for (2.26) and (2.27) respectively. Hence, there is a contradiction: (2.26) and (2.27) cannot be satisfied whilst $a_i = a_j = a$ and both lines coincide. ■

Since (2.26) and (2.27) do not coincide, it must be the case that they cross only once. Hence, an equilibrium exists and it must be unique. The same reasoning also demonstrates a unique equilibrium exists for the case of observable and verifiable effort.

Equilibrium Conditions for the Two-Firm Case

Moral Hazard

In the subsection above, (2.25) gives the FOC for profit maximisation by firm i. Re-arranging (2.25) and assuming a symmetric equilibrium, the equilibrium condition becomes:

$$D - 5a^7\tau^4 - 20a^6\tau^3 - 30a^5\tau^2 - 20a^4\tau - 5a^3 - Da\tau = 0$$

First Best

When effort is observable and verifiable, the expected profit function for firm i becomes:

$$E(\pi_i) = \frac{Da_i}{(\tau a_i + 1)^2 (\tau a_j + 1)} - \frac{1}{4}a_i^4$$

and the FOC is:

$$\frac{\partial E(\pi_i)}{\partial a_i} = \frac{D(1 - \tau a_i)}{(\tau a_i + 1)^3 (\tau a_j + 1)} - a_i^3 = 0$$

Re-arranging and applying symmetry, the equilibrium condition for a is:

$$D - a^7\tau^4 - 4a^6\tau^3 - 6a^5\tau^2 - 4a^4\tau - a^3 - Da\tau = 0$$

Maximisation of Joint Profits

If the firms act as a single monopolist their joint profit maximisation problem is:

$$\max_{a_i,a_j} E(\pi_{i+j}) = \frac{Da_i}{(\tau a_i + 1)^2 (\tau a_j + 1)} + \frac{Da_j}{(\tau a_j + 1)^2 (\tau a_i + 1)} - \frac{1}{4}a_i^4 - \frac{1}{4}a_j^4$$

The two FOCs are:

$$\frac{\partial E(\pi_{i+j})}{\partial a_i} = \frac{D(1 - 2a_i a_j \tau^2 - a_i\tau)}{(\tau a_i + 1)^3 (\tau a_j + 1)^2} - a_i^3 = 0$$

$$\frac{\partial E\left(\pi_{i+j}\right)}{\partial a_j}=\frac{D\left(1-2a_ia_j\tau^2-a_j\tau\right)}{\left(\tau a_j+1\right)^3\left(\tau a_i+1\right)^2}-a_j^3=0$$

Applying symmetry, these conditions reduce to:[39]

$$D-a^8\tau^5-5a^7\tau^4-10a^6\tau^3-10a^5\tau^2-5a^4\tau-a^3-2Da^2\tau^2-Da\tau=0$$

Equilibrium Conditions for Three- and Four-Firms Cases

These equilibrium conditions are obtained using a procedure equivalent to that for the two-firm case.

Three Firms

First Best:

$$D-a^8\tau^5-5a^7\tau^4-10a^6\tau^3-10a^5\tau^2-5a^4\tau-a^3-Da\tau=0$$

Moral Hazard:

$$D-5a^8\tau^5-25a^7\tau^4-50a^6\tau^3-50a^5\tau^2-25a^4\tau-5a^3-Da\tau=0$$

Joint Profit Maximisation:

$$D-a^{10}\tau^7-7a^9\tau^6-21a^8\tau^5-35a^7\tau^4-35a^6\tau^3-21a^5\tau^2$$

$$-7a^4\tau-3Da^3\tau^3-a^3-5Da^2\tau^2-Da\tau=0$$

[39]Due to the functional forms of $\frac{\partial E(\pi_{i+j})}{\partial a_i}$ and $\frac{\partial E(\pi_{i+j})}{\partial a_j}$, for now, assume that the problem is strictly quasiconcave.

Four Firms

First Best:

$$D - a^9\tau^6 - 6a^8\tau^5 - 15a^7\tau^4 - 20a^6\tau^3 - 15a^5\tau^2 - 6a^4\tau - a^3 - Da\tau = 0$$

Moral Hazard:

$$D - 5a^9\tau^6 - 30a^8\tau^5 - 75a^7\tau^4 - 100a^6\tau^3 - 75a^5\tau^2 - 30a^4\tau - 5a^3 - Da\tau = 0$$

Joint Profit Maximisation:

$$D - a^{12}\tau^9 - 9a^{11}\tau^8 - 36a^{10}\tau^7 - 84a^9\tau^6 - 126a^8\tau^5 - 126a^7\tau^4$$

$$-84a^6\tau^3 - 36a^5\tau^2 - 4Da^4\tau^4 - 9a^4\tau - 11Da^3\tau^3 - a^3 - 9Da^2\tau^2 - Da\tau = 0$$

2.8.6. Appendix 2.6 - Equilibrium Conditions When $f(q_i|k, a_i)$ is Used

Let the probability density function for output be:

$$f(q_i|k, a_i) = \left\{ \begin{array}{cc} \frac{1}{ka_i}e^{-\frac{q_i}{ka_i}}, & q_i \geq 0 \\ 0, & q_i < 0 \end{array} \right\}$$

Using this probability density function and setting $R = 0$, but otherwise using a model identical to that in section 2.3, the conditions for the equilibrium effort level each principal will induce are:

First Best	$\frac{1}{ka^2}e^{-\frac{B}{ka}}\left(\frac{1}{3}B^3+5k^3a^3+\frac{3}{2}B^2ka+4Bk^2a^2\right)+Bk-5k^2a-a^3=0$
Moral Hazard	$\frac{1}{ka^2}e^{-\frac{B}{ka}}\left(\frac{1}{3}B^3+5k^3a^3+\frac{3}{2}B^2ka+4Bk^2a^2\right)+Bk-5k^2a-5a^3=0$
Max. Joint Profits	$\frac{1}{2ka^2}e^{-\frac{B}{ka}}\left(B^3+4B^2ka+10Bk^2a^2+12k^3a^3\right)+Bk-6k^2a-a^3=0$

[40]

Lemma A2.14 *When effort is unobservable, or unverifiable, and the probability density function is given by $f(q_i|k,a_i)$, the condition for a symmetric equilibrium is:*

$$\frac{1}{ka^2}e^{-\frac{B}{ka}}\left(\frac{1}{3}B^3+5k^3a^3+\frac{3}{2}B^2ka+4Bk^2a^2\right)+Bk-5k^2a-5a^3=0$$

Proof. Solve principal i's problem in two steps, as in section 2.4. Firstly, find the cost-minimising contract for principal i to induce an effort a_i. Secondly, identify the effort level, a_i, that principal i should induce to maximise profits given the cost-minimising incentive contract.

Using the results from section 2.4, the cost-minimising incentive contract for the principal's problem described in (2.4) must satisfy:

$$(w_i(q_i))^{\frac{1}{2}}=\lambda_i+\mu_i\frac{f_{a_i}(q_i|k,a_i)}{f(q_i|k,a_i)}$$

Inserting the expressions for $f(q_i|k,a_i)$ and $f_{a_i}(q_i|k,a_i)$ gives:

$$w_i(q_i)=\left(\lambda_i+\mu_i\left(\frac{q_i-ka_i}{ka_i^2}\right)\right)^2 \tag{2.32}$$

[40] Setting $k=1$, these conditions reduce to those stated in section 2.5.1.

Inserting (2.32) into the ICC and the PC, noting $R = 0$, and then solving as a system of two equations in two unknowns gives:

$$\lambda_i = \frac{a_i^2}{2}$$

$$\mu_i = a_i^3$$

Inserting these values for λ_i and μ_i back into (2.32) gives the cost-minimising contract to induce the effort a_i as:

$$w_i^*(q_i) = \frac{1}{4}\left(a_i^2 + \frac{2a_i}{k}(q_i - ka_i)\right)^2$$

Significantly, the expected wage cost, $E(w_i)$, is independent of k:

$$E(w_i) = \int_0^{\infty} \frac{1}{4}\left(a_i^2 + \frac{2a_i}{k}(q_i - ka_i)\right)^2 \frac{1}{ka_i} e^{-\frac{q_i}{ka_i}} dq_i = \frac{5}{4}a_i^4$$

The intuition for this result is that when the probability density function is changed the optimal incentive contract, $w_i^*(q_i)$, also changes. Hence, the change in the probability density function is offset by the form of the incentive contract to leave the expression for the expected wage cost unchanged.[41]

In contrast, changing the probability density function does alter the expected revenue function. Holding a_j fixed, the second stage of principal i's problem is to maximise the following expected profit function:

[41] Investigating the generality of this result is left for future research.

$$\max_{a_i} E(\pi_i) = E(R_i) - E(w_i^*) =$$

$$k^2 a_i a_j^3 \frac{e^{-\frac{B}{ka_j}}}{(a_i - a_j)^2} + k a_i^2 e^{-\frac{B}{ka_i}} \frac{Ba_i - Ba_j + 2ka_i^2 - 3ka_i a_j}{(a_i - a_j)^2} + ka_i (B - 2ka_i - ka_j) - \frac{5}{4} a_i^4$$

Assuming strict quasiconcavity, the FOC for principal i to maximise expected profits is:

$$\frac{\partial E(\pi_i)}{\partial a_i} = -k^2 a_j^3 \frac{e^{-\frac{B}{ka_j}}}{(a_i - a_j)^3} (a_i + a_j)$$

$$+ \frac{e^{-\frac{B}{ka_i}}}{(a_i - a_j)^3} \begin{pmatrix} B^2 a_i^2 - 2B^2 a_i a_j + B^2 a_j^2 + 3Bka_i^3 - 8Bka_i^2 a_j \\ +5Bka_i a_j^2 + 4k^2 a_i^4 - 11k^2 a_i^3 a_j + 9k^2 a_i^2 a_j^2 \end{pmatrix}$$

$$-4k^2 a_i - k^2 a_j + Bk - 5a_i^3 = 0$$

Appealing to the problem's symmetry, combining the first two terms, and applying l'Hôpital's rule three times, gives the condition for a symmetric equilibrium stated in Lemma A2.14. ■

Lemma A2.15 *When effort is observable and verifiable and the probability density function is given by $f(q_i|k, a_i)$, the condition for a symmetric equilibrium is:*

$$\frac{1}{ka^2} e^{-\frac{B}{ka}} \left(\frac{1}{3} B^3 + 5k^3 a^3 + \frac{3}{2} B^2 ka + 4Bk^2 a^2 \right) + Bk - 5k^2 a - a^3 = 0$$

Proof. When effort is observable and verifiable the expected revenue function is identical to that in the proof of Lemma A2.14. As effort is observable and verifiable, a forcing contract can be used as described in the proof of Lemma A2.7. Since effort is observable, the agent is exposed to no risk under the forcing contract and so the form of the probability density function does not affect the expected wage cost. The expected cost of inducing an effort a_i is simply:

$$E\left(w_i^*\right) = \frac{1}{4}a_i^4 \tag{2.33}$$

As a result, the unconstrained profit maximisation problem facing principal i is:

$$\max_{a_i} E\left(\pi_i\right) = k^2 a_i a_j^3 \frac{e^{-\frac{B}{ka_j}}}{\left(a_i - a_j\right)^2}$$

$$+ka_i^2 e^{-\frac{B}{ka_i}} \frac{Ba_i - Ba_j + 2ka_i^2 - 3ka_i a_j}{\left(a_i - a_j\right)^2} + ka_i\left(B - 2ka_i - ka_j\right) - \frac{5}{4}a_i^4$$

From here, following the steps described in the proof of Lemma A2.14, it is straightforward to obtain the equilibrium condition described in Lemma A2.15. ■

Lemma A2.14 *When effort is observable and verifiable, firms maximise joint profits and the probability density function is given by* $f(q_i|k, a_i)$*, the condition for a symmetric equilibrium is:*

$$\frac{1}{2ka^2}e^{-\frac{B}{ka}}\left(B^3 + 4B^2ka + 10Bk^2a^2 + 12k^3a^3\right) + Bk - 6k^2a - a^3 = 0$$

Proof. As effort is observable and verifiable, the expression for the expected wage cost is given by (2.33). This means the problem faced jointly by the two principals is:

$$\max_{a_i, a_j} E\left(\pi_{i+j}\right) = E\left(R_i\right) + E\left(R_j\right) - \frac{1}{4}a_i^4 - \frac{1}{4}a_j^4$$

where

$$E(R_i) + E(R_j) = \frac{k}{a_i - a_j}\left(e^{-\frac{B}{ka_i}} a_i^2 (B + 2ka_i) - e^{-\frac{B}{ka_j}} a_j^2 (B + 2ka_j)\right)$$

$$+ka_i (B - 2ka_i - ka_j) + ka_j (B - 2ka_j - ka_i)$$

There are two FOCs for this problem, $\frac{\partial E(\pi_{i+j})}{\partial a_i} = 0$ and $\frac{\partial E(\pi_{i+j})}{\partial a_j} = 0$. The FOC with respect to a_i is:

$$\frac{\partial E(\pi_{i+j})}{\partial a_i} = \frac{1}{(a_i - a_j)^2}\left(\begin{array}{c} e^{-\frac{B}{ka_i}} (4k^2 a_i^3 + B^2 a_i - B^2 a_j - 6k^2 a_i^2 a_j + 3Bka_i^2 - 4Bka_i a_j) \\ +e^{-\frac{B}{ka_j}} ka_j^2 (B + 2ka_j) \end{array}\right)$$

$$-4k^2 a_i - 2k^2 a_j + Bk - a_i^3 = 0$$

Again, from here, using the steps described in the proof of Lemma A2.14, the equilibrium condition stated in Lemma A2.16 is obtained. ■

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CHAPTER 3

The Economics of the Criminally Inclined

3.1. Introduction

Most people choose not to commit crime[1]; however, some people do. As a starting point, one might expect those desperate for money and with less to lose, such as the unemployed, to have a greater likelihood of offending. A well functioning social security system hopefully helps to reduce the incentives of the unemployed to commit crime. This chapter considers the criminal choice in a dynamic optimisation framework where agents are heterogeneous. The optimal choice of crime and job search is essentially a portfolio decision problem, which depends on an agent's tastes and opportunities. We also identify a link between unemployment, crime and gambling, even though the utility of consumption is assumed to be strictly concave. For certain agent types, whom we refer to as the "criminally inclined", gambling (say in a fair game of poker) yields strictly positive value. The model then provides a framework to understand the associations between personal characteristics, economic circumstances and self-reports of offending in an unusually rich dataset: the Offending, Crime and Justice Survey (OCJS), 2003-2006. In this dataset, covering England and Wales, an intuitive proxy for "integrity" is found to have a statistically significant negative relationship with the probability of offending. However, respondents' employment status

[1]When we refer to crime we focus solely on economic crime. We define economic crime as an activity deemed illegal by society which leads to monetary benefit and/or makes extra non-monetary assets available for consumption. The more limited definition of the variable "Economic Crime" used in the empirical analysis is provided in Table 3.1 of section 3.7.2.

and their self-assessments of financial position do not show consistently significant relationships with offending. Whilst the lack of relationship between employment status and offending is surprising, the theoretical model offers a number of explanations for this result.

Viewing the criminal choice as a portfolio decision problem can be understood in the following way: committing economic crime, such as shoplifting, yields an instant financial pay-off but carries the risk of arrest and future time spent in jail. In contrast, job search while unemployed has the opposite structure: it is a costly investment made today whose financial return is deferred to the future (it takes time to find employment). An additional feature of the real world is incomplete insurance: a thief cannot purchase insurance against the risk of jail and an unemployed worker cannot purchase insurance against failing to find work. The optimal criminal choice is therefore the solution to a dynamic forward-looking decision problem based on an assessment of risks.

The heterogeneous agents differ regarding: (i) their labour market characteristics, such as wages earned, employment status, job search costs and expected duration of unemployment etc., (ii) their wealth[2] and (iii) their aversion to (disutility from) committing crime, a characteristic we refer to as "integrity". Given the assumption of rational decision making, many insights are immediate. For example, as one is not allowed to consume out of savings whilst in jail, going to jail has a higher opportunity cost for the rich. As such, a career in crime is an "inferior good" and one indulged

[2]A liquidity constraint requires agents' asset holdings to be non-negative.

in by the relatively poor. Similarly, a high wage worker has more to lose by going to jail and so has a reduced incentive to commit crime. At first glance, this statement suggests that, on average, the employed will commit less crime.

A central insight is that, depending on tastes and opportunities, agents sort (or self-select) into criminal behaviour or otherwise. Given such sorting, an interesting issue is how many individuals switch into and out of crime over time. If relatively few switch between crime and no-crime strategies over the business cycle, this would suggest the responsiveness of crime rates to cyclical changes in unemployment may be small in magnitude.

An agent who commands a high wage in the labour market and has high integrity will have little interest in committing crime while unemployed. If laid off, their optimal strategy is to invest in job search to find new employment and use a dissavings strategy to self-insure against the low income stream received whilst unemployed. Conversely, agents with low integrity and who can only earn, say, the minimum wage whilst (legally) employed, have a comparative advantage in "crime".[3] These low-integrity agents sort into criminal behaviour. Significantly, these "criminally inclined" agents may be just as likely to commit crime while employed and earning low wages as while unemployed and on benefits.

[3] See Burdett et al (2003, 2004).

Despite the initial intuition that, on average, the employed will commit less crime, the OCJS data shows that the group reporting the highest offending rate[4] is those in routine and manual occupations. It is the high offending rate amongst these respondents which drives the surprising result that the offending rates for Theft and Economic Crime are higher for the employed than for those looking for work.[5] To explain this, firstly, note that workers in this group are probably low paid and experience poor working conditions. Hence, the difference in their utility when employed and unemployed may be small.[6] The result is also explained by the high prevalence of workplace theft recorded. Once one controls for workplace and school theft, the offending rate of those looking for work is higher than for those employed in intermediate or higher occupations.

Additionally, that the survey period 2003-2006 was a period of benign economic conditions is important. It appears even "criminally inclined" individuals could find employment during this period.

Of course, there will be agents who do switch between committing crime whilst unemployed and not committing crime whilst employed. We refer to these types as “unfortunates”. Again, the benign economic conditions when the OCJS was conducted probably meant that the number of unemployed "unfortunates" was small.

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[4]In this chapter, the term offending rate refers to a percentage, calculated as the number of observations displaying a particular characteristic and where the respondent offended, divided by the total number of observations displaying the relevant characteristic.

[5]The variable "Theft" represents all theft including vehicle theft, theft from work, theft from school, robbery and burglary (although there are few observations of these latter two crimes). "Economic Crime" is defined as Theft plus selling drugs, selling stolen goods and credit card fraud. Full details of the sample and offence categories are provided in section 3.7, whilst further detail about the employment status question is given in Table 3.13. All of the analysis uses a sub-sample of the OCJS data. The sub-sample covers respondents aged 17-25.

[6]Any difference in utility was probably further reduced, for the vast majority of respondents, as they lived with their parents. As such, transfers within family units may have provided an additional, informal, form of unemployment insurance.

In the model, those with an integrity high enough to never commit crime behave according to a standard job search model - the option to commit crime has no value. This chapter's novel contribution is the description of optimal dynamic behaviour by those agents with sufficiently low integrity that they are willing to commit crime. We identify three criminal types. These types share one common feature: each will commit crime when unemployed, but only when their liquidity constraint binds.

One criminal type has such a low return to labour that they never look for work, are permanently unemployed and always commit crime. These agents spend their lives in and out of jail. Being inactive in the labour market, their criminal activity is largely immune to business cycle variations in unemployment.

The "unfortunates" are more interesting. When unemployed and with a positive stock of assets, they use an optimal dissavings strategy to smooth consumption over time. If their asset stock is not too high, they will also search for employment. Only when their assets are exhausted do they switch to crime. However, even when this occurs they continue to look for work and, on finding employment, will stop committing crime.

The most interesting criminal type is the "criminally inclined". These agents search for jobs when unemployed, but will continue to commit crime when employed, if they have no assets. This criminal type also has non-standard financial incentives: when unemployed, these agents obtain a surplus by gambling in fair lotteries even though their utility is strictly concave.[7] Gambling is optimal for this type, when unemployed, because it allows specialisation. If a "criminally inclined" agent gambles heavily and wins big, i.e. achieves a threshold level of assets, then, on finding employment, the agent goes straight and never commits crime again. If, instead, the agent loses everything so they have no assets, they immediately switch to a life of crime. For intermediate asset levels a smooth dissavings strategy while unemployed is not optimal. An unemployed "criminally inclined" agent with an intermediate level of assets will buy lottery tickets in the hope of a big win and, to maximise the probability of winning, will bet their total stock of assets. If they lose their shirt, they immediately switch to crime.

The OCJS data is consistent with this result. Figures 3.1 shows that those who favour risk are more likely to report offending. Also, offenders like taking risks.[8]

The positive value of gambling to the "criminally inclined" provides an additional explanation for the empirical link between gambling venues and increases in crime after their opening.[9] Not only risk-lovers, but also the "criminally inclined" will be drawn to locations where there are opportunities to gamble.

[7]This non-convexity issue also arises in the optimal unemployment insurance literature where unemployed individuals follow optimal job search and savings strategies: see, for example, Kocherlakota (2004), Booth and Coles (2007), Lentz and Tranaes (2005).
[8]For additional detail see section 3.7.2.
[9]See Grinols and Mustard (2006) and Wheeler et al (2011).

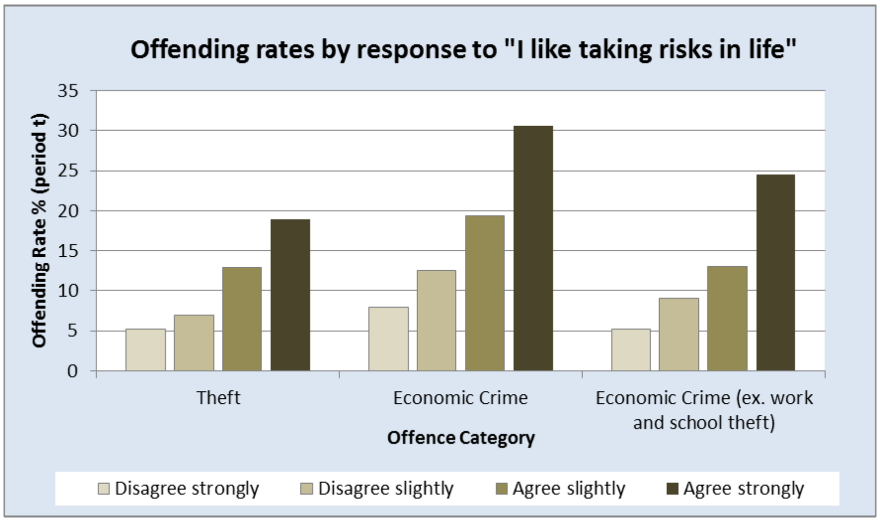

Figure 3.1: Offending rates in period t by attitude to risk at the end of period t-1.

The importance of "integrity" in identifying agent types drove the selection of the OCJS dataset. To the best of our knowledge, the OCJS is unique in allowing a comparison of individuals' attitudes towards breaking the law (a clear proxy for integrity) and subsequent offending. Figure 3.2 shows the strong positive association between our chosen measure of integrity and subsequent offending.

The strength of association between this integrity proxy and offending is confirmed by probit models of offending. In the preferred specification[10], an attitude shift from "Agree" to "Strongly disagree" is associated with a statistically significant average reduction in a respondent's offending probability of up to 9.9 percentage points.

[10] See Specification 1 in Table 3.6.

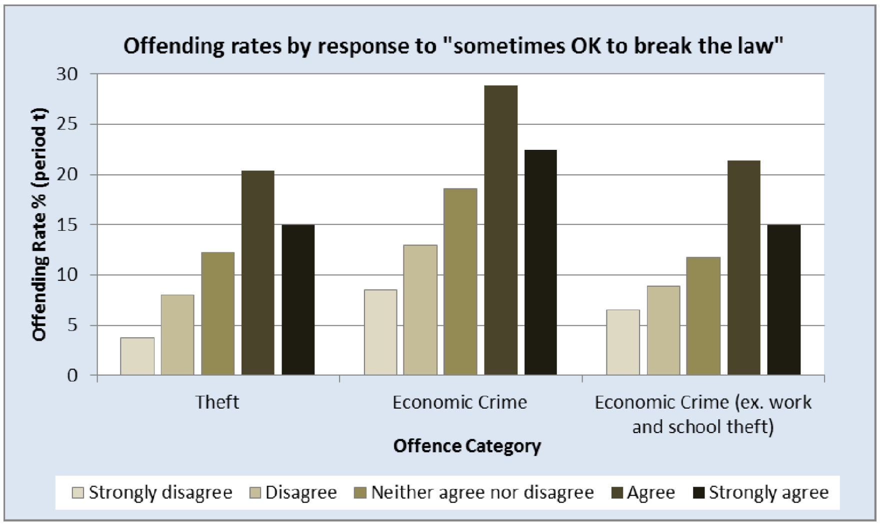

Figure 3.2: Offending rates in period t by attitude to breaking the law at first inteview.

The OCJS data also enables a control which proxies peer effects. The marginal effect of the integrity proxy reported above is robust to the inclusion of this control. Nevertheless, having friends in trouble with the police (our peer effects proxy) is associated with a statistically significant average increase in a respondent's offending probability of between 5.2 and 7.8 percentage points.

The variable which noticeably reduces the statistical significance of the integrity proxy's average marginal effects is a control for prior offending. However, the strength of association between prior offending and subsequent offending reports still supports the notion of agents specialising in crime. Previous offending can be interpreted as an additional signal of low integrity. Reporting an offence prior to first interview is associated with a statistically significant average increase of up to 12.2 percentage points in the offending probability.

The rest of the chapter comprises two parts. Sections 3.2 to 3.5 present the theoretical model and consider the optimal crime, job search, gambling and savings strategies of workers for a range of individual characteristics. Sections 3.6 to 3.9 use the theoretical model as a framework to analyse the OCJS data.[11] Section 10 concludes.

3.2. Theoretical Literature

Early theoretical contributions on the economics of crime include Becker (1968), Ehrlich (1973) and Block and Heineke (1975). These models emphasise the cost-benefit nature of the criminal decision with individuals comparing the expected benefits of crime against the expected costs of punishment. Whilst these papers do not specify the labour market in detail, they do highlight the importance of the earnings differential between legal and illegal sources of income in determining criminal activity. To some extent, all three papers, and in particular Block and Heinke (1975), also note the potential influence of "psychic" costs of crime, or individuals' varying aversions to committing crime. Thus, the need to accommodate integrity into economic models of crime has long been recognised.

More recently, Conley and Wang (2006) incorporate an individual's aversion to crime into a sorting model. Here, individuals choose a level of education to obtain and make a binary choice between legal employment and criminal activity. Individuals with lower integrity[12] and lower ability specialise in criminal activity.[13]

[11]Sections 3.3 to 3.5 are the work of Prof. Melvyn Coles, whilst Sections 3.6 to 3.9 are my work.
[12]Conley and Wang use the term "honesty".
[13]Fender (1999) also includes a simple notion of integrity by dividing the population he considers into "incorruptibles" who never commit crime and "corruptibles" whose criminal decision depends on the wage available.

The paper that introduced a criminal decision into a search theoretic model of the labour market was Burdett et al (2003). In contrast to the present chapter, Burdett et al (2003) develop an equilibrium model of the labour market. However, the present chapter is complementary, as it offers a significant increase in the complexity of the agent's decision problem. Whilst Burdett et al (2003) consider ex-ante identical workers, in our model there is significant agent heterogeneity. Also, our agents have to determine the optimal saving/dissaving strategy in the presence of liquidity constraints.

Engelhardt (2010) develops a search model incorporating agent heterogeneity regarding agents' flow utility whilst unemployed. Engelhardt finds that if this flow utility is sufficiently high, an agent will never commit crime due to the opportunity cost of jail. This result - that only a sub-section of the population commit crime - is similar to our model. However, as with Burdett et al, Engelhardt (2010) does not include an optimal savings problem with a liquidity constraint into his model.

Another search theoretic model is Engelhardt et al (2008). This paper adapts Pissarides (2000) to incorporate a criminal decision and an optimal employment contract. This model is then calibrated, using US data, to analyse the relative impacts of labour market policies and criminal justice policies in determining crime rates. Also, Huang et al (2004) considers the interplay of human capital investment with the legal and criminal sectors. Depending on the level of education obtained, individuals specialise in either legal or criminal activity.[14]

[14]Other theoretical papers linking the labour market and crime, but not involving search, are Lochner (2004) and İmrohoroğlu, Merlo and Rupert (2000, 2004).

3.3. The Model

The model extends the standard job search framework in continuous time where $t \in [0, \infty)$. Consider a representative agent who is infinitely lived, discounts the future at rate $r > 0$ and is characterised by the following parameters:

(i) the integrity parameter $k \geq 0$ describes the agent's (flow) disutility to committing crime;

(ii) if, while unemployed, the agent searches for a job with effort s, then λs describes the rate at which the agent receives a job offer, while ds describes the agent's flow disutility to search. Assuming search effort must be finite, there is no further loss in generality by assuming s is a binary choice variable $s \in \{0, 1\}$;[15]

(iii) w describes the market wage the worker enjoys once employed.

The agent obtains flow utility $u(c)$ from consumption $c \geq 0$, where $u(.)$ is a strictly increasing and strictly concave function. Each agent uses an optimal savings strategy where $A \geq 0$ denotes the agent's wealth and r also describes the market interest rate. There is a liquidity constraint: having no collateral when $A = 0$, the poor are unable to borrow from banks. As agents are liable to commit crime, and so go to jail, when $A = 0$, this crime margin reinforces the banks' decision not to lend.

There are incomplete insurance markets: the agent cannot insure against re-employment risk, nor against the risk of conviction. While unemployed, an agent

[15]Given linear costs and continuous time, the worker can search with effort $s = 1$ for a fraction σ of the next instant $dt > 0$, and so effectively searches with effort $\sigma \in [0, 1]$ at cost $\mathrm{d}\sigma dt$. Setting $s = 1$ as the upper bound is equivalent to re-normalising λ and d.

receives a constant social security benefit b. On re-employment, we simplify the problem by assuming a job is for life. Hence, employed workers do not have a precautionary savings motive.

As pointed out in the introduction, some "criminal" agents would like to gamble in fair lotteries. However, for the most part, gambling does not generate a positive return in the optimal crime/job search/savings strategy. For ease of exposition, we largely ignore the potential purchase of lottery tickets. Instead, we introduce this possibility only when it becomes relevant, i.e. when describing the optimal behaviour of the "criminally inclined".[16]

The agent can be in one of three states: $i \in \{J, U, E\}$ corresponding to being in jail, being unemployed and being employed. If not in jail, each agent can choose a criminal activity level $z \geq 0$ where z describes the resulting flow income from crime. Given current criminal activity z, $\gamma z dt$ describes the probability of being convicted over the next instant $dt > 0$. In an extended equilibrium framework, one might assume γ depends on police resources and on aggregate criminal activity. In this version, however, we fix γ as a parameter.

The agent is sent to prison if convicted of criminal activity; i.e. γ describes the conviction rate per unit of crime. During a prison spell, a prisoner cannot consume any of their savings. Instead he/she obtains a given flow utility u_J and simply waits until release. The prison spell is described by an exponential distribution with parameter μ. Hence $\frac{1}{\mu}$ describes the expected jail-term. Although μ potentially could be conditioned

[16]See Section 3.5.2.

on the level of crime committed, for simplicity, we assume μ is a constant.[17] On release from jail, the agent returns to the labour market as an unemployed individual.

We next describe the Bellman equations for the value functions in each state $i \in \{J, U, E\}$. The solution to these value functions depends on the agent's wealth, A, (a state variable) and their fixed characteristics $X = \{k, \lambda, d, w, b\}$. As X is held fixed throughout, we simplify notation by subsuming reference to X in the value functions below.

3.3.1. When In Jail

As a convicted individual with wealth A is given a jail term distributed according to an exponential distribution with parameter μ, the expected value of being convicted is:

$$V^J(A) = \frac{u_J + \mu V^U(A)}{r + \mu} \tag{3.1}$$

where, on release, the worker is unemployed with value $V^U(A)$. For simplicity, it is assumed the agent's assets, A, are frozen while in jail (perhaps hidden under the floorboards). As we shall show that agents only indulge in criminal activity when liquidity constrained, i.e. when $A = 0$, this assumption only involves a minor loss of generality.

[17] As in the light bulb example used to motivate Poisson processes, the court only observes that the light bulb has gone out, not the likelihood with which it was going to expire.

3.3.2. When Unemployed

At each point in time, the unemployed worker chooses consumption $c \geq 0$, criminal activity $z \geq 0$ and job search effort $s \in \{0, 1\}$ to maximise expected lifetime value. While unemployed, the agent's savings evolve according to:

$$\dot{A} = rA + b + z - c$$

Thus, given current assets A, the Hamilton/Jacobi/Bellman equation describing privately optimal behaviour while unemployed is:

$$rV^U(A) = \max_{\substack{c,z\geq 0 \\ s\in\{0,1\}}} \begin{bmatrix} u(c) - kz - ds + \frac{dV^U}{dA}\left[rA + b + z - c\right] \\ +z\gamma\left[V^J(A) - V^U(A)\right] + s\lambda\left[V^E(A) - V^U(A)\right] \end{bmatrix} \tag{3.2}$$

subject to the constraint $A \geq 0$. $V^E(A)$ describes the agent's value from being employed with assets A. The integrity parameter, k, describes the agent's disutility from performing an illegal act while $d > 0$ describes the disutility of time spent looking for work.

3.3.3. When Employed

At each point in time, an employed agent chooses consumption $c \geq 0$ and criminal activity $z \geq 0$ but, as all firms pay the same wage w, we assume no on-the-job search and set $s = 0$. While employed, the agent's savings evolve according to:

$$\dot{A} = rA + w + z - c$$

Given current assets, A, the Hamilton/Jacobi/Bellman equation describing privately optimal behaviour while employed is:

$$rV^E(A) = \max_{c,z \geq 0} \begin{bmatrix} u(c) - kz + \frac{dV^E}{dA}\left[rA + w + z - c\right] \\ +z\gamma\left[V^J(A) - V^E(A)\right] \end{bmatrix} \tag{3.3}$$

subject to the constraint $A \geq 0$.

3.3.4. Preliminary Comments and Insights

Describing optimal behaviour requires jointly solving the above Bellman equations for $V^i(.)$. The decision rules for the optimal choice of $\{c, s, z\}$ are functions of the state variable, A, and the underlying characteristics, X. The solution to these Bellman equations is non-trivial as insurance is incomplete: the optimal choice of $\{c, s, z\}$ depends on the mix of risks associated with the chosen portfolio of actions.

The simplifying assumption that the returns to crime are linear in z is empirically useful. If, instead, the cost of crime function, $k(z)$, were strictly convex with the Inada condition $k'(0) = 0$, all agents would commit a small amount of crime. The advantage of linear returns is that, consistent with the data, most citizens choose not to commit any crime. The central interest, of course, is understanding the interaction between job search incentives, criminal behaviour and the consumption choice.

The assumption of no lay-off risk once employed is critical for analytical tractability. It implies an employed agent has no precautionary motive to save. This, in turn, ensures the wealth state $A = 0$ is absorbing: when unemployed with $A = 0$ an agent is liquidity constrained (unable to borrow further) and when employed with $A = 0$ an agent has no incentive to save for the future. Solving the Bellman equation for each $V^i(.)$ is then straightforward: we first characterise the optimal choice of $\{c, z, s\}$ and the corresponding $V^i(.)$ at $A = 0$. Given that solution, we can then iterate backwards to identify the optimal strategies for $A > 0$. Introducing lay-off risk would instead require computing these value functions numerically. As it is unlikely that adding lay-off risk per se would significantly change the model's insights, beyond marginally reducing the value of employment, we exclude this possibility and obtain analytical results.

This structure yields the following simplifications. First, we show that in the optimal solution, *no agent ever commits crime when* $A > 0$. The intuition for this is that an agent cannot consume out of wealth A whilst in jail, and this foregone consumption option implies a richer agent has a lower return to crime. Thus, the poor agent has a "comparative advantage" in committing crime relative to his/her wealthier self. The linear returns to crime then ensure all agents delay criminal activity until $A = 0$.

Second, an income gap $b < w$ ensures that it is strictly better to be employed than unemployed. As the agent has less to lose through committing crime when unemployed then, *if it is ever optimal to commit crime, the worker will commit crime when unemployed with* $A = 0$. Conversely, we show that if it is not optimal to commit crime when unemployed with $A = 0$, it is never optimal to commit crime. We classify this latter class of agents as "honest". Furthermore, as the option to commit crime generates no surplus for "honest" agents, their behaviour reduces to that of a standard job search model (with savings).

The complementary group of "dishonest" agents, who commit crime when unemployed with $A = 0$, is our primary interest. A sufficiently large wage gap $w-b$, ensures these agents do not commit crime when employed (they have too much to lose). As employment is then an absorbing state, it follows straightforwardly that an agent consumes permanent income $w + rA$ while employed and so $V^E(A) = \frac{u(w+rA)}{r}$ for any $A \geq 0$. Given this solution for $V^E(.)$, it is relatively straightforward to characterise $V^U(.)$ and so describe job search and crime for this type of agent.

Life is much more complicated, and more interesting, for "dishonest" agents whose wage gap, $w-b$, is sufficiently small that the agent will commit crime when employed if $A = 0$, and whose search costs are sufficiently low that an unemployed agent with $A = 0$ will seek employment. The tension is that the agent is better off when employed, as $w > b$, but employment is no longer an absorbing state. At some point in time, the agent will be convicted and, after a prison spell, will be unemployed. This suggests that an employed agent has a precautionary savings motive: to accumulate savings while employed to self-insure against going to jail and subsequently being unemployed. However, this cannot describe optimal behaviour. Once an employed agent has accumulated $A > 0$, it is no longer optimal for them to commit crime. If they do not commit crime, then there is no risk of jail and, in turn, no precautionary savings motive.

The surprising result is that these agents wish to use gambling strategies while unemployed. We refer to these agents as "criminally inclined" and, given the non-standard nature of their optimal behaviour, we analyse this type separately (see section 3.5.2). Nevertheless it is important to note that, even for this type, we show $A = 0$ remains an absorbing state.

3.4. Optimal Job Search and Crime when $A = 0$ is an Absorbing State

Anticipating that $A = 0$ is an absorbing state, we first solve for the value functions $V^i(0)$ and find the corresponding optimal choices of $\{c, z, s\}$. The subsequent section uses backward iteration to characterise these functions and decision rules for all $A \geq 0$. Of course, we then verify that the solution to the Bellman equations does imply $A = 0$ is an absorbing state.

When unemployed and liquidity constrained with $A = 0$, consumption equals $b+z$, where z is the agent's crime rate in this state. Similarly, consumption while employed is $w + z$. Using (3.1) to substitute out $V^J(0)$, the Bellman equations (3.2) and (3.3), describing the values of being unemployed and employed with $A = 0$, reduce to:

$$rV^U(0) = \max_{\substack{z\geq 0 \\ s\in\{0,1\}}} \left[\begin{array}{c} u(b+z) - z\left(k + \gamma[\frac{rV^U(0)-u_J}{r+\mu}]\right) \\ +s\left(\lambda\left[V^E(0) - V^U(0)\right] - d\right) \end{array} \right] \tag{3.4}$$

and

$$rV^E(0) = \max_{z\geq 0} \left[u(w+z) - z\left(k + \gamma\left[V^E(0) - \frac{u_J + \mu V^U(0)}{r+\mu}\right]\right)\right] \tag{3.5}$$

(3.4) and (3.5) are a closed pair of recursive equations for $V^U(0)$ and $V^E(0)$.

Define the No Crime Constraint, NCC, as the parameter values X where the unemployed worker with characteristics X, and $A = 0$, is just indifferent to committing crime. From (3.4), the NCC is identified by:

$$(NCC) \qquad u'(b) = k + \gamma[\frac{rV^U(0) - u_J}{r + \mu}]$$

where, in extended notation, $V^U(.) = V^U(.|X)$. Note the LHS of the NCC describes the marginal return to crime, whilst the RHS describes its marginal cost. Agents with sufficiently high integrity, i.e. those with $k \geq u'(b) - \gamma[\frac{rV^U(0) - u_J}{r+\mu}]$, do not commit crime when unemployed with $A = 0$. As we show such agents never commit crime, agents with integrity on or above the NCC are labelled "honest".

Agents with integrity below the NCC commit crime when unemployed and liquidity constrained. An important distinction, however, is that some of these agents also commit crime when employed. Define the No Crime Constraint (Employed), NCC_E, as the parameter values X such that an employed agent with $A = 0$ is indifferent to committing crime. From (3.5), this constraint is identified by:

$$(NCC_E) \qquad u'(w) = k + \gamma\left[V^E(0) - \frac{u_J + \mu V^U(0)}{r + \mu}\right]$$

As $w > b$ guarantees it is better to be employed than unemployed, i.e. $V^E(0) > V^U(0)$, it follows that the NCC_E lies below the NCC in (k, w) space. Those with integrity between these constraints are classified as "unfortunates": once employed they stop committing crime as they then have too much to lose. In contrast, the "criminally inclined" - those with integrity below the NCC_E - commit crime even when employed.

The Job Search constraint, JS, in Figure 3.3 is defined as the parameter values X for which an unemployed agent with $A = 0$ is indifferent between $s \in \{0, 1\}$. From (3.4), this corresponds to the condition:

$$(JS) \qquad V^E(0) - V^U(0) = \frac{d}{\lambda}$$

Although only implicit in this equation, this constraint identifies a critical wage threshold where, ceteris paribus, an agent strictly prefers $s = 1$ for wages above the threshold.

A closed form solution for this partition requires solving for the endogenous values $V^i(0)$. To illustrate, consider the frictionless limit $\lambda \to \infty$. In this limit, an agent with $w > b$ chooses $s = 1$ and immediately finds work. For such w, the closed form solution for NCC_E is:

$$(NCC_E) \qquad k = u'(w) - \frac{\gamma}{r + \mu}\left[u(w) - u_J\right]$$

Note the marginal return to committing crime in this state is $u'(w)$, while the marginal loss includes the integrity cost k and the expected loss from conviction.

Again for $w > b$, which ensures job search is incentive compatible, the NCC has the closed form solution:

$$(NCC) \qquad k = u'(b) - \frac{\gamma}{r+\mu}\left[u(w) - u_J\right]$$

This time, when unemployed, the marginal return to committing crime is $u'(b)$ but, as the agent expects to be earning the wage w in the (very) near future, the marginal loss from conviction continues to depend on w. Both of these constraints are downward sloping, they intersect at $w = b$ and the NCC_E is below the NCC for all $w > b$.

For $w < b$, the agent does not look for work and so is a member of the long-term unemployed. The NCC in this case reduces to:

$$k = u'(b) - \frac{\gamma}{r+\mu}\left[u(b) - u_J\right]$$

as the worker expects to live on benefits, b, indefinitely and optimally selects to commit no crime, $z = 0$.

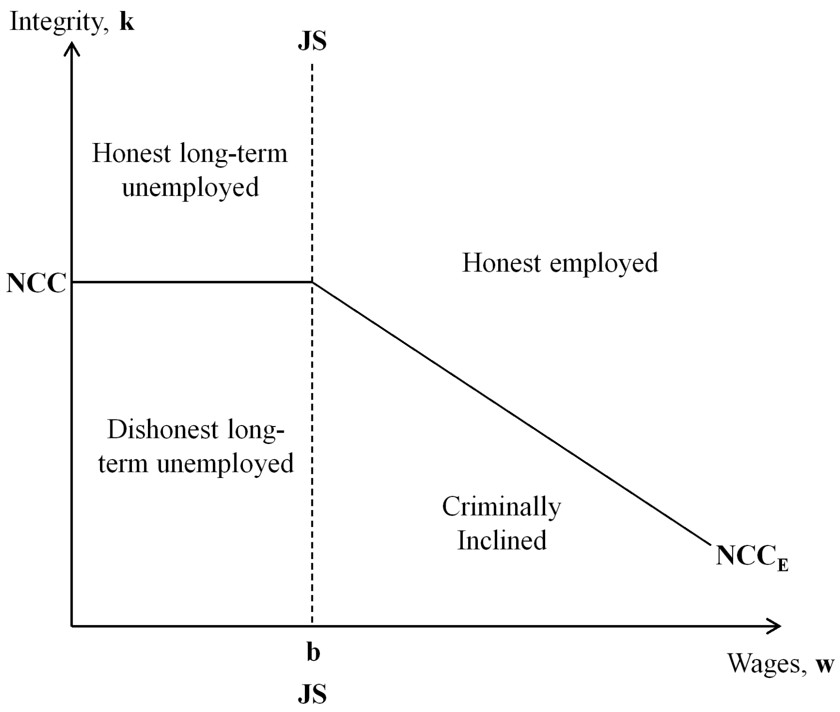

Figure 3.3: Agent types observed when job search frictions are absent.

As such, the frictionless limit identifies five possible types of behaviour when $A = 0$:

(i) **"honest" job seekers** who choose $s = 1$ and never commit crime;

(ii) **"unfortunates"** who choose $s = 1$ and only commit crime when unemployed;

(iii) the **"criminally inclined"** who choose $s = 1$ and commit crime both when employed and unemployed;

(iv) the **"honest" long-term unemployed** who choose $s = 0$ but live honestly on benefits b;

(v) the **"criminal" long-term unemployed** who choose $s = 0$ and commit crime.

In the frictionless limit, the number of unfortunates who commit crime is zero: they each find work arbitrarily quickly and do not commit crime when employed. In the frictionless limit, all crime is committed by the criminal long-term unemployed and by the criminally inclined. Hence, in Figure 3.3 only four types of behaviour are shown.

It is interesting to consider how the level of benefits, b, affects the structure of crime and unemployment. An increase in b shifts the JS constraint to the right and increases the set of long-term unemployed individuals. The NCC, however, shifts down and so there is an unambiguous increase in the number of "honest" individuals. Thus an increase in benefits reduces crime, but at the cost of increasing long-term unemployment.

Finally, note that the ability to earn a higher wage not only moves an agent out of long-term unemployment; it also switches an agent from being criminally inclined to being an "honest" job seeker. The worker switches away from crime once the value of employment is sufficiently high.

Even with labour market frictions, $\lambda < \infty$, the structure of this partition remains largely intact. It is easy to show that no agent has an incentive to look for work whenever $u(w) < u(b) + \frac{rd}{\lambda}$. For such types, the NCC is identified as:

$$k = u'(b) + \gamma \frac{u_J - u(b)}{r + \mu}$$

This is the same as before: for the long-term unemployed who choose $s = 0$, the return to crime does not depend on λ.

For $u(w) > u(b) + \frac{rd}{\lambda}$, active job search is potentially optimal. Whether the agent chooses to look for work, however, depends on their integrity, k. In essence, the unemployed agent is choosing between a portfolio of risky options: to seek employment at flow cost d (to obtain future wage, w, though such a position is only reached at rate λ) and/or to commit crime (which pays z immediately but incurs the cost of imprisonment at rate γz). The optimal portfolio choice depends on the agent's integrity, k, and the wage earned while employed, w.

A little algebra establishes the NCC is now given by:

$$\text{(NCC)} \qquad k = u'(b) - \frac{\gamma}{r+\mu}\left[u(b) - u_J + \frac{\lambda}{r+\lambda}\left[u(w) - u(b) - \frac{rd}{\lambda}\right]\right]$$

This condition is slightly more complicated than before as, whilst unemployed, the job seeker finds employment at rate λ, and $u(w) - u(b) - \frac{rd}{\lambda}$ describes the flow surplus whilst employed. However, the interpretation for the NCC is unchanged. The only difference is the cost of conviction now includes the foregone option value of looking for work. The NCC remains a downward sloping function of w. The intuition is that an increase in w raises the value of being employed which, at the NCC margin, causes the agent to switch away from crime as the loss from conviction is now too high. Thus, along the NCC, an increase in w causes the criminal to substitute from crime to legal employment; i.e. crime and job search are substitute activities.

It is straightforward to obtain an explicit solution for the JS constraint. For "honest" agents, i.e. those above the NCC, the JS constraint is identified by $u(w) = u(b) + \frac{rd}{\lambda}$. Agents with a potential wage above this threshold are active job seekers; the others are long-term unemployed. This threshold does not depend on k as "honest" agents always choose $z = 0$.

For the "dishonest" agents, who lie below the NCC, the expression for the JS constraint is very long and not particularly helpful. The key insight, as depicted in Figure 3.4, is that the JS constraint is downward-sloping for criminal agents. Thus along the JS constraint, an increase in integrity, k, would cause a criminal to invest in job search.

We establish this result using the Envelope Theorem. For "dishonest" agents with $A = 0$, let $z^U > 0$ denote the optimal crime rate when unemployed and $z^E \geq 0$ denote the optimal crime rate when employed. A useful result when $w > b$ (established in Proposition 3.1) is that $z^U > z^E \geq 0$; i.e. the "criminally inclined" choose a lower level of crime when employed. By the Envelope Theorem and for the parameter values X on the JS constraint, the Bellman equations (3.4) and (3.5) mean that an increase in integrity, k, implies:

$$r\frac{dV^U(0)}{dk} = -z^U - \frac{\gamma r}{(r+\mu)}\frac{dV^U(0)}{dk}$$

$$r\frac{dV^E(0)}{dk} = -z^E - \gamma z^E\left[\frac{dV^E(0)}{dk} - \frac{\mu}{r+\mu}\frac{dV^U(0)}{dk}\right]$$

As $z^U > z^E \geq 0$, simple algebra now establishes $\frac{dV^E(0)}{dk} > \frac{dV^U(0)}{dk}$; i.e. an increase in integrity has a greater downward impact on the value of being unemployed than on the value of being employed. This is largely because the agent commits more crime whilst unemployed. This implies the JS constraint is downward-sloping for "dishonest" agents: an increase in integrity increases the return to search, as $\frac{d}{dk}[V^E(0) - V^U(0)] > 0$, and so the wage earned whilst employed must fall to ensure the agent remains indifferent to job search. With market frictions, crime and job search are substitute activities: as integrity increases, the unemployed agent chooses less crime and switches to active job search.

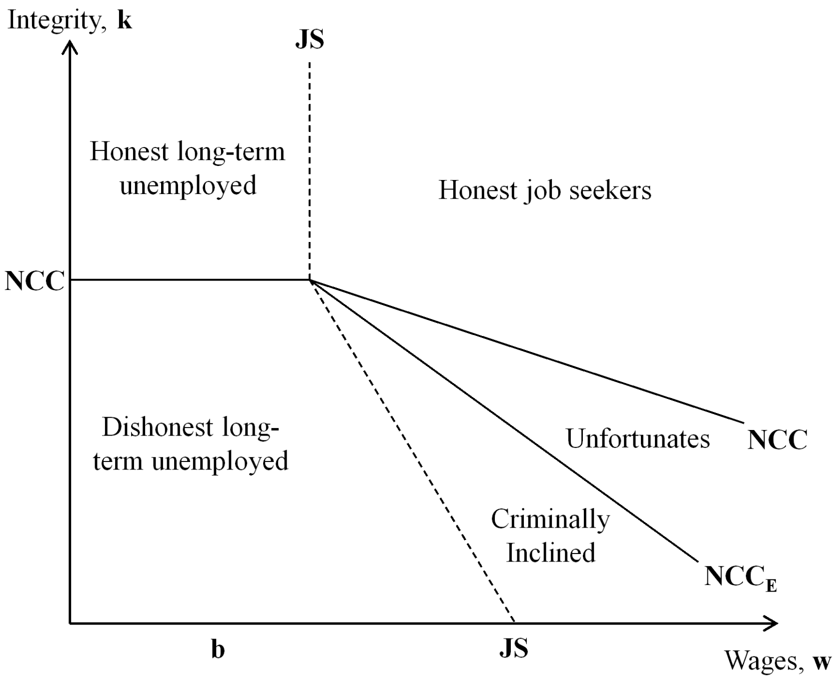

Figure 3.4: Agent types when job search frictions are present.

Note for wages, such that $u(w) > u(b) + \frac{rd}{\lambda}$, the NCC for active job seekers, given above, depends directly on the expected duration of unemployment, $\frac{1}{\lambda}$. An increase in the expected duration of unemployment (lower λ) shifts the NCC upwards and reduces the number of "honest" job seekers. A lower return to job search (it takes longer to find work) leads agents to switch to crime.

3.5. Optimal Savings Strategies when $A > 0$

The previous section described optimal behaviours when $A = 0$, for each possible type X. This section now uses an induction argument to describe optimal behaviours for all $A \geq 0$.

It is obvious that $V^U(.)$ is strictly increasing in A. Consider the Bellman equation (3.2), which describes the value of being unemployed with assets $A \geq 0$. As $u(.)$ is strictly concave, the optimal consumption choice is given by the standard first-order condition (FOC):

$$u'(c) = \frac{dV^U}{dA}$$

The solution of this FOC implies the optimal consumption rule $c = c^U(A)$.

As the return to search effort, s, is linear, optimality implies:

(JS Condition) $$s = 1 \text{ if } V^E(A) - V^U(A) \geq \frac{d}{\lambda}$$

where we assume a job seeker who is indifferent between $s = 0$ and $s = 1$ chooses $s = 1$. The jobless look for employment only if the return from doing so exceeds its cost. Below, we show this structure yields a critical asset level, A^P, where only the sufficiently poor, i.e. those with $A \leq A^P$, choose $s = 1$. Of course, this asset level A^P depends on agent characteristics X.

Substituting out $V^J(A)$ from the Bellman equation given by (3.2) implies that for any $A > 0$, the unemployed worker optimally chooses $z = 0$ when:

$$k > \frac{dV^U}{dA} + \gamma \left[\frac{u_J - rV^U(A)}{r + \mu} \right] \tag{3.6}$$

However, note that if crime whilst unemployed, $z^U > 0$, is optimal when $A = 0$, the optimal choice is given where:

$$u'(b + z^U) + \gamma \left[\frac{u_J - rV^U(0)}{r + \mu} \right] = k$$

This condition implies (3.6) only holds with equality at $A = 0$. Thus as long as $V^U(.)$ is an increasing concave function, then, if (3.6) holds with equality when $A = 0$, (3.6) must hold with strict inequality for all $A > 0$; i.e. crime is never optimal for $A > 0$. However, somewhat surprisingly, it is not immediate that $V^U(.)$ is concave. Indeed, the analysis is problematic for the "criminally inclined". Hence, we consider this case separately.

The Bellman equation (3.3) describes the value of being employed. The optimal consumption choice implies:

$$u'(c) = \frac{dV^E}{dA}$$

the solution of which gives the optimal consumption rule, $c = c^E(A)$. The return to criminal activity is linear, and the agent prefers not to commit crime whilst employed with $A > 0$ whenever:

$$k > \frac{dV^E(A)}{dA} + \gamma \left[V^J(A) - V^E(A) \right] \tag{3.7}$$

Given agent characteristics X and the corresponding solution for $V^U(.)$, $V^E(.)$ at $A = 0$, all that remains is to apply backward induction from this solution, using the optimal control rules described above. As the solution is standard for "honest" agents, we focus on the two most interesting cases, the "unfortunates" and the "criminally inclined". As the solutions are very different, we consider each case separately.

3.5.1. Optimal Behaviour for the "Unfortunates" ($A \geq 0$)

Fix parameter values X consistent with being an "unfortunate". Thus at $A = 0$, job search, $s = 1$, committing crime when unemployed, $z^U > 0$, and not committing crime whilst employed, $z^E = 0$, are all optimal. Given these choices, the payoffs V^U and V^E are determined by (3.4) and (3.5).

Now consider $A > 0$. Suppose for the moment that, given the characteristics X, crime is never optimal when employed. As employment is then an absorbing state, the agent optimally consumes permanent income, $c^E = w + rA$, and so:

$$V^E(A) = \frac{u(w + rA)}{r}$$

Given this conjectured solution for $V^E(.)$, we now characterise the corresponding solution for $V^U(.)$. We then verify in the proof of Theorem 3.1 that, for these parameters X, (3.7) is satisfied for all $A \geq 0$, implying that not committing crime, $z = 0$, is indeed optimal when employed. Hence, the expression $V^E(.)$ above solves the Bellman equation (3.3).

Now consider an "unfortunate" who is unemployed with $A > 0$. The previous section identifies an initial value for $c^U(0) = b + z^U$. The obvious approach is to identify the optimal consumption strategy, $c^U(.)$, given this initial value, whilst noting that $V^U(.)$ is the solution to the initial value problem:

$$\frac{dV^U}{dA} = u'(c^U(A)),$$

with the initial value $V^U(0)$ given by (3.4). It is important to recognise that, if consumption $c^U(.)$ increases with wealth, A, the value function $V^U(.)$ is necessarily concave. This latter result then establishes that committing crime, $z > 0$, is never optimal for $A > 0$.

Using the optimal consumption rule $u'(c) = \frac{dV^U}{dA}$ and the Envelope Theorem then, whilst $s = 1$ is optimal, the Bellman equation (3.2) implies the agent's optimal consumption smoothing strategy evolves according to the pair of differential equations:

$$[-u''(c)]\dot{c} = \lambda[u'(w + rA) - u'(c)] \tag{3.8}$$

$$\dot{A} = rA + b - c \tag{3.9}$$

(3.8) describes the optimal consumption smoothing strategy when the agent finds employment at rate λ, at which point the marginal utility of consumption falls to $u'(w + rA)$. The optimal strategy is forward looking: the "unfortunate" takes into account that at $A = 0$ he/she becomes liquidity constrained and consumes $c^U(0) = b + z^U$. Formally, the optimal consumption strategy $c^U(.)$ is the solution to the above dynamic system with the initial value $c^U(0) = b + z^U$. Figure 3.5 provides the corresponding phase diagram when $z^U < w - b$.

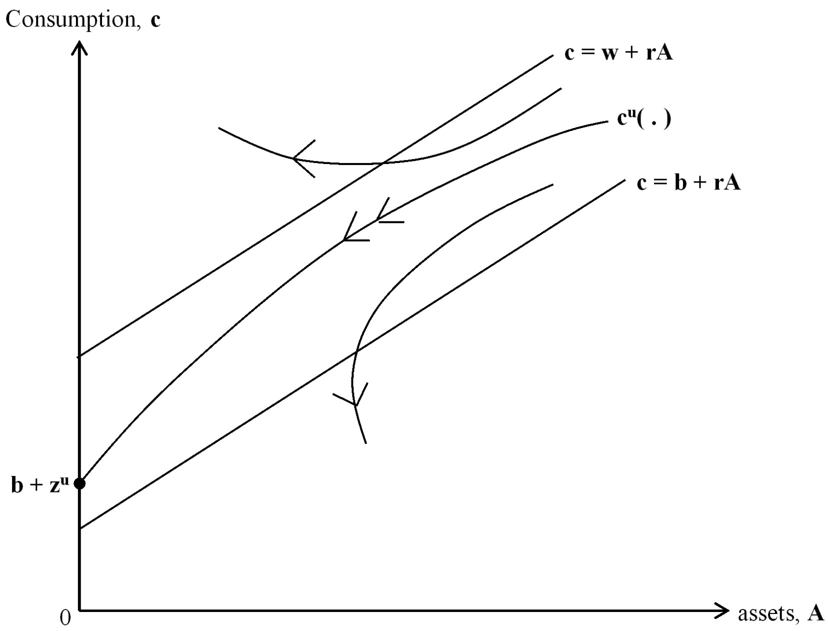

Figure 3.5: Phase diagram showing the optimal consumption strategy for an "unfortunate".

Whilst job search, $s = 1$, is optimal, a simple contradiction argument, using Figure 3.5, establishes the optimal consumption rule satisfies $c^U(A) \in (b + rA, w + rA)$ for all A and is a strictly increasing function. If $c^U(A)$ was not a strictly increasing function, the trajectory could not converge to the initial value $c^U(0)$ as $A \to 0$. Note

$c^U(A) > b + rA$ implies assets fall over time: the job seeker uses a dissaving strategy to reduce the consumption gap between c^U when unemployed and $c^E = w + rA$ when employed. Once $A = 0$, the worker is liquidity constrained and switches to crime, $z^U > 0$, to consume $c^U(0) = b + z^U$.

Of course, the phase diagram in Figure 3.5 only applies whilst $s = 1$ is optimal, which, in turn, requires $V^E(A) - V^U(A) \geq \frac{d}{\lambda}$. This inequality is satisfied at $A = 0$ since, by definition, the characteristics X of "unfortunates" imply $s = 1$ is optimal at this point. Furthermore, as optimal consumption, $c^U(A) < c^E(A) = w + rA$, the return to search, $V^E(A) - V^U(A)$, is continuous and strictly decreasing in A when $s = 1$ is optimal. Thus, there exists a critical asset level, say $A = A^P$, at which point $V^E(A) - V^U(A) = \frac{d}{\lambda}$. This asset level identifies the active job search region. For $A \in [0, A^P]$, the unemployed worker chooses $s = 1$ and, as consumption $c^U(.)$ is a strictly increasing function, it follows that $V^U(.)$ is strictly concave over this region. This confirms it is optimal not to commit crime, $z = 0$, in this region.

For $A > A^P$, we continue the induction process, noting that $s = 0$ and $z = 0$ are optimal in this range. Optimal consumption smoothing now implies $\dot{c} = 0$; i.e. c^U remains constant over time. As V^U is (weakly) concave, it follows that the no crime constraint continues to hold. Additionally, $c^U < c^E$ implies that $V^E(A) - V^U(A)$ continues to decrease as A increases and so $s = 0$ remains optimal. We now have enough information to complete the description of optimal behaviour for the "unfortunates".

Theorem 3.1: Optimal Behaviour of the "Unfortunates"

For characteristics X consistent with being an "unfortunate", whose optimal crime when unemployed is $z^U < w - b$, the optimal strategy is:

(1) Crime: the agent never commits crime except when unemployed and liquidity-constrained; i.e. when $A = 0$;

(2) Job search: the agent chooses $s = 1$ when $A \leq A^P$;

(3) Consumption when employed: the agent consumes their permanent income $c^E = w + rA$, which is an absorbing state.

(4) Consumption when unemployed:

(i) for low $A \in [0, A^P)$, consumption, $c^U(.)$, is strictly increasing in A and exceeds $b + rA$ so that assets fall over time;

(ii) for $A \in [A^P, A^R)$, where $A^R = \frac{c^U(A^P)-b}{r}$, consumption, $c^U = c^U(A^P)$, does not change with A but again exceeds $b + rA$ so that assets fall over time;

(iii) for $A \geq A^R$ the agent consumes permanent income $c^U = b + rA$, which is an absorbing state.

Proof. See the Technical Appendix.

This induction approach also applies when characteristics X are consistent with being an "honest" job seeker; i.e. someone who chooses $s = 1$ and $z^U = 0$ at $A = 0$. The phase diagram in Figure 3.5 continues to apply; the only difference is that the initial consumption value is now $c^U(0) = b$. The same argument as above applies: the optimal consumption smoothing strategy implies $c^U(.)$ is an increasing function of wealth. As this implies $V^U(.)$ is a concave function, it follows that crime is never optimal while unemployed. As the agent has even more to lose when employed, the agent also does not commit crime while employed. Finally, note that $A = 0$ is an absorbing state: when unemployed the agent is liquidity constrained and cannot borrow further, and when employed the agent consumes permanent income $c^E = w$. This approach thus identifies the solution to the Bellman equations.

An important feature of Theorem 3.1 is that it restricts attention to $z^U < w - b$. If instead $b + z^U > w$, then consumption whilst unemployed, $c^U(0) = b + z^U$, exceeds the wage earned when employed. Therefore, in this case, the agent has an incentive to also commit crime when employed. We now show that such agents, the "criminally inclined", have very different savings incentives.

3.5.2. Optimal Behaviour for the "Criminally Inclined" ($A \geq 0$)

From now on, we assume the presence of fair lotteries and show that the "criminally inclined" enjoy a strictly positive return from gambling. Of course, the presence of such lotteries ensures $V^U(.)$ is (weakly) concave. This, in turn, ensures that crime is never optimal for $A > 0$.

Fix parameter values X consistent with being "criminally inclined". Thus at $A = 0$, job search, $s = 1$, committing crime whilst unemployed, $z^U > 0$, and committing crime whilst employed, $z^E > 0$, are all optimal. The Bellman equations (3.4) and (3.5) imply the values $V^E(0)$ and $V^U(0)$ and the optimal crime rates z^E and z^U are jointly determined by:

$$(3.10) \qquad rV^E(0) = u(w+z^E) - kz^E + z^E\gamma \left[V^J(0) - V^E(0)\right]$$

$$(3.11) \qquad u'(w+z^E) = k + \gamma \left[V^E(0) - V^J(0)\right]$$

$$(3.12) \qquad rV^U(0) = \begin{array}{c} u(b+z^U) - kz^U - d \\ +z\gamma \left[V^J(0) - V^U(0)\right] + \lambda \left[V^E(0) - V^U(0)\right] \end{array}$$

$$(3.13) \qquad u'(b+z^U) = k + \gamma \left[V^U(0) - V^J(0)\right]$$

with $V^J(0)$ given by (3.1).

It is not surprising that the "criminally inclined" commit more crime when unemployed. Proposition 3.1, however, shows they commit significantly more crime when unemployed.

Proposition 3.1. *"Criminally inclined" agents with* $A = 0$ *choose* $z^U > z^E + w - b$.

Proof: The criminally inclined have $V^E(0) > V^U(0)$ since $s = 1$ is optimal. Equations (3.11) and (3.13) then imply $u'(w+z^E) > u'(b+z^U)$ which yields Proposition 3.1. ■

Having less to lose when unemployed, the crime rate of the "criminally inclined" when unemployed implies they actually consume more than when employed; i.e. $c^U(0) = b + z^U$ exceeds $c^E(0) = w + z^E$. As $w - b$ is typically small for the "criminally inclined" (see Figure 3.4), the difference in crime rates when employed and unemployed may not be particularly large. Nevertheless, this yields a non-standard result: an agent's marginal utility of consumption is higher when employed than when unemployed. Not surprisingly, this generates non-standard financial incentives.

The essential intuition for what follows is that crime and job search are substitute activities. Committing crime reduces the return to job search (being convicted implies a worker loses their job), while being employed reduces the return to crime (a worker has more to lose). Being substitute activities, an agent would prefer to specialise. The solution to the Bellman equations centres around an endogenously determined wealth level, denoted $A^S > 0$, such that an agent will never commit crime when employed with $A = A^S$. It is not optimal to accumulate this asset level A^S through crime. Instead, the "criminally inclined" attempt to win A^S through gambling.

For $A \in [0, A^S]$, where A^S is determined in Theorem 3.2 below, an unemployed agent uses the following gambling strategy: they bet all their assets so that a win yields wealth level A^S, while a loss yields zero wealth. A fair lottery implies they win with probability $p = \frac{A}{A^S}$. Thus, for such an A, the value of being unemployed is:

$$V^U(A) = V^U(0) + \frac{A}{A^S}\left[V^U(A^S) - V^U(0)\right],$$

which is linear and increasing in A. Furthermore, optimality of z^U at $A = 0$ requires:

$$u'(b + z^U) = \frac{dV^U(0)}{dA} = \frac{\left[V^U(A^S) - V^U(0)\right]}{A^S}, \tag{3.14}$$

while linearity of the value function over $[0, A^S]$ further implies $c^U(A^S) = b + z^U$.

In the optimal solution, the employed agent with $A \geq A^S$ never commits crime, consumes permanent income $c^E = w + rA$ and so obtains the value $V^E(A) = \frac{u(w+rA)}{r}$. Now consider the unemployed agent with $A \geq A^S$ but A small enough that $s = 1$ remains optimal. The agent's optimal consumption smoothing strategy again is described by the differential equations (3.8) and (3.9), but this time with the initial value $c^U = b + z^U$ at $A = A^S$. Further, the proof of Theorem 3.2 below establishes that optimality requires $w + rA^S > b + z^U$. Figure 3.6 portrays the relevant phase diagram for the optimal consumption strategy.

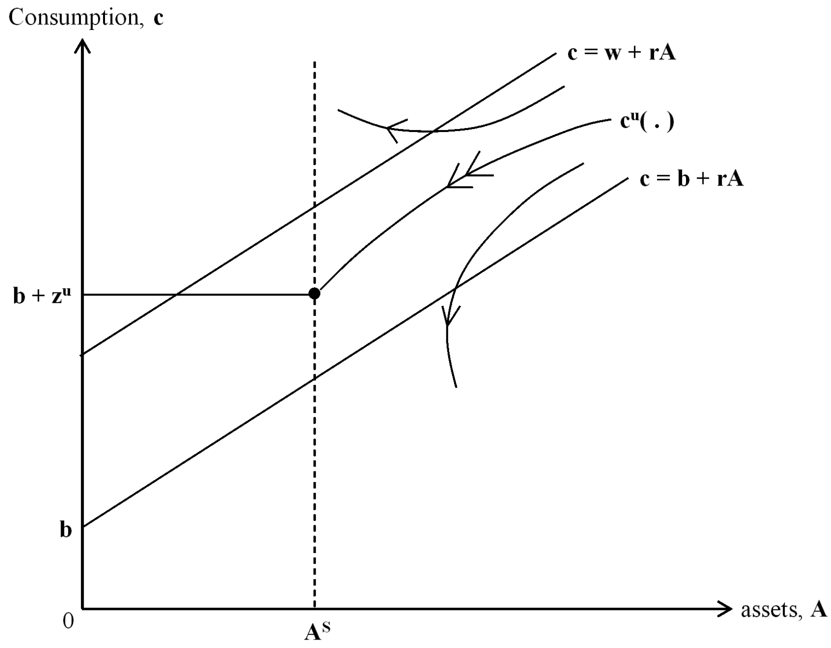

Figure 3.6: Phase diagram showing the optimal consumption strategy for a "criminally inclined" individual.

Whilst $A \geq A^S$, but A is small enough that $s = 1$ remains optimal, then, as before, optimal consumption is $c^U(A) \in (b + rA, w + rA)$ and assets fall over time. Once assets fall to the critical threshold $A^S > 0$, the agent consumes $c^U = b + z^U$ but, as consumption exceeds income $b+rA$, the agent has to finance this income shortfall. At $A = 0$, this shortfall is financed by switching to crime. At A^S, however, the shortfall is now financed through gambling. The job seeker bets their remaining wealth A^S which, in a fair lottery, is lost at a Poisson rate α such that $\alpha A^S = c^U(A^S) - b - rA^S$. We can give an explicit example supposing a fair roulette wheel. Over each (small) time period $\Delta > 0$, the agent bets $(c^U(A^S) - b - rA^S)\Delta$ on red. If they win, they walk away with their winnings and their assets are successfully maintained at A^S. If they lose, they double their bet. Whenever they win they walk away and the net winnings cover their income shortfall, $(c^U(A^S) - b - rA^S)\Delta$. Of course, they keep doubling their bet every time they lose and, with probability $\alpha\Delta$, they lose everything. In the limit, as $\Delta \to 0$, this gambling strategy maintains wealth at A^S but the agent loses everything according to a Poisson process with parameter $\alpha = \frac{z^U}{A^S} - r$. Once penniless, the agent switches to crime, $z^U > 0$.

This gambling strategy yields the value:

$$rV^U(A^S) = u(b + z^U) - d + \lambda \left[\frac{u(w + rA^S)}{r} - V^U(A^S) \right]$$

$$+\alpha[V^U(0) - V^U(A^s)]$$

the solution of which is:

$$V^U(A^S) = \frac{u(b + z^U) - d + \alpha V^U(0) + \lambda \frac{u(w+rA^S)}{r}}{r + \lambda + \alpha}$$

Using this expression to substitute out $V^U(A^S)$ in (3.14), and noting $\alpha = \frac{z^U}{A^S} - r$, yields the following equation for A^S:

$$u(b + z^U) - d - (r + \lambda)V^U(0) - z^U u'(b + z^U) = \lambda u'(b + z^U)A^S - \frac{\lambda}{r} u(w + rA^S)$$

As (3.12)-(3.13) imply:

$$(r + \lambda)V^U(0) = u(b + z^U) - d - z^U u'(b + z^U) + \lambda V^E(0)$$

substituting out $V^U(0)$ in the previous expression yields:

$$\text{(3.16)} \qquad \frac{u(w + rA^S)}{r} - A^S u'(b + z^U) = V^E(0)$$

Remarkably, this condition is equivalent to:

$$V^E(A^S) - V^U(A^S) = V^E(0) - V^U(0)$$

Hence, as job search, $s = 1$, is optimal at $A = 0$, it is also optimal at $A = A^S$.

The final step is to show that a solution for A^S exists, is unique, implies $w + rA^S > b + z^U$ (as depicted in Figure 3.6); and that when employed with $A \geq A^S$ the optimal strategy is never to commit crime. The proof of Theorem 3.2 in the Technical Appendix establishes this result.

Theorem 3.2: Optimal behaviour of the criminally inclined

The optimal strategy of a "criminally inclined" agent is:

(1) Crime: $z = 0$ for all $A > 0$, but, at $A = 0$, $z = z^U > 0$ and $z = z^E > 0$ as identified by the solution to (3.10)-(3.13);

(2) Gambling while unemployed: for $A \in [0, A^S]$ the worker bets everything where, in the event of a win, the agent holds wealth $A = A^S$;

(3) Optimal job search: $s = 1$ when $A < A^P$ where $A^P > A^S$;

(4) Consumption whilst unemployed:

(i) for $A \leq A^S$, the worker consumes $c^U = b + z^U$;

(ii) for intermediate asset levels $A \in [A^S, A^P]$, consumption $c^U(.)$ is strictly increasing in A and exceeds $b + rA$ so that assets fall over time;

(iii) for $A \in [A^P, A^R)$*, where* $A^R = \frac{c^U(A^P)-b}{r}$*, consumption* $c^U = c^U(A^P)$ *does not change with* A *but exceeds* $b + rA$ *so again assets fall over time;*

(iv) for $A \geq A^R$*, the worker consumes permanent income* $c^U = b + rA$ *which is an absorbing state;*

(5) Consumption while employed:

(i) for assets $A < \frac{z^E}{r}$ *, the worker consumes* $c^E = w + z^E$ *and, as assets fall over time, switches to crime when* $A = 0$*;*

(ii) for assets $A \geq \frac{z^E}{r}$*, the worker goes straight and consumes permanent income* $c^E = w + rA$*. As* $A^S > \frac{z^E}{r}$*, the employed worker with* $A = A^S$ *goes straight.*

Proof. See the Technical Appendix.

Finally, note that $A = 0$ is indeed an absorbing state. Hence, the above solution method is applicable.

3.6. Existing Empirical Evidence

The theoretical model now provides a framework to analyse data from the Offending, Crime and Justice Survey (OCJS). Before describing the OCJS data, the relationship between the theoretical model and existing empirical results is discussed.

3.6.1. Unemployment and Crime

A large empirical literature explores the link between unemployment and economic crime. Studies from the US consistently find a statistically significant link between unemployment and economic crimes. However, there is debate regarding whether changes in unemployment rates are sufficient to explain the drop in property crime seen during the 1990s. Levitt (1996, 1997 and 2004), along with Donohue and Levitt (2001), consistently find an elasticity of around 1 between a percentage point change in the unemployment rate and percentage changes in the crime rate. Hence, Levitt (2004) argues that the 2 percentage point drop in the US unemployment rate between 1991 and 2001 was insufficient to explain the 28.8% drop in property crime over the same period. In contrast, other authors, including Raphael and Winter-Ebmer (2001), Gould et al (2002), Lin (2008) and Mocan and Bali (2010), report higher elasticities of crime with respect to unemployment. For example, Mocan and Bali (2010) find that a 1 percentage point increase in the unemployment rate increases the crime rate by 2-4%.[18]

Also concerning the US, Engelhardt (2010) structurally estimates a search model of the labour market which incorporates crime. Using individual-level data from the National Longitudinal Survey of Youth 79 (NLSY79), Engelhardt estimates that the incarceration rate for the unemployed is double that for low-wage workers and quadruple that for high-wage workers.

Turning to Europe, almost all studies find a statistically significant link between aggregate unemployment rates and economic crime. Using a panel of European countries, Altindag (2012) finds a significant positive relationship between unemployment and economic crime. Similarly, Fougere et al (2009), Edmark (2005) and Öster and

[18] Also, Mocan and Bali (2010) find property crime responds asymmetrically to unemployment changes across the business cycle. Crime is more sensitive to unemployment during periods of rising unemployment.

Agell (2007) all find significant positive relationships between unemployment and economic crimes. Fougere et al (2009) consider youth unemployment in France, whilst Edmark (2005) and Öster and Agell (2007) both consider Swedish data.

Using panels of UK police force areas (PFAs), Witt et al (1999) and Carmicheal and Ward (2001) find a significant positive relationship between the unemployment rate, or changes in the unemployment rate, and crime. However, Machin and Meghir (2004) fail to find a statistically significant link between unemployment and crime once PFA fixed effects are considered.

Whilst all of these results, apart from Machin and Meghir (2004), are in contrast to our empirical findings, they are consistent with the theoretical model when $w - b > 0$. Also, as many of these studies cover longer time periods than the OCJS, they can pick up business cycle fluctuations and include periods of higher unemployment.

3.6.2. Wages, Benefits and Crime

In the present model, when wages (benefits) are increased, the NCC_E (NCC) is met at lower values of k. Holding the distribution of k fixed, we would then expect a negative relationship between wages (benefits) and economic crime. This finding matches the empirical results. Grogger (1998) finds a negative relationship between the log of wages and economic crime using data from the NLSY79. The same relationship, again using US data, is also found by Gould et al (2002) and Mocan and Unel (2011).

Turning to England and Wales, Machin and Meghir (2004) find a negative relationship between wages at the 25th percentile in the wage distribution and economic crimes. Using a difference-in-difference estimation strategy which compares PFAs, Hansen and Machin (2002) show the introduction of the minimum wage in 1999 reduced economic crime rates. These findings fit with the evidence provided in the introduction that those in low-level occupations show the highest offending rates.

Switching to the role of benefits, Machin and Marie (2006) find that the introduction of the Job Seekers' Allowance in 1996, with its tougher eligibility criteria, led to increased economic crime. Lastly, Feinstein and Sabates (2008) find that the introduction of the educational maintenance allowance for 16-18 year olds, when combined with improved policing initiatives, was associated with a drop in burglaries.

3.6.3. Asset Holdings, Financial Constraints and Crime

More limited empirical research exists on the direct role of asset holdings and liquidity constraints in determining criminal behaviour.

Probably the most interesting work is Foley (2011). Foley compares daily reports of crimes in twelve US cities and considers their relationship to the monthly cycle of welfare payments. In cities where welfare payments occur at the start of each month an increase in crime is recorded towards the end of each month. This temporal crime pattern does not occur in cities where welfare payments are staggered across the month. The present model explains this temporal variation by viewing each welfare payment as an endowment of assets, A. Individuals only commit crime once A has been exhausted, i.e. towards the end of the month.[19]

A number of other papers also provide some evidence of a relationship between liquidity constraints and crime. However, they either show mixed results or do not, themselves, argue that binding liquidity constraints cause individuals to commit crime. For example, Morse (2011) argues that payday lenders helped to mitigate increases in shoplifting following natural disasters in California. Also, Garmaise and Moskowitz (2006) show that neighbourhoods containing less banking competition had higher interest rates and subsequently experienced higher economic crime rates. However, Immergluck and Smith (2006) fail to find a statistically significant relationship between the foreclosure rate in Chicago neighbourhoods and economic crime.

Lastly, McIntyre and Lacombe (2012) consider data from London in 2004-2005 on county court judgements (CCJs). CCJs are issued when an individual has difficulties paying off debt. These authors find a statistically significant relationship between the total value of CCJs issued within a neighbourhood and robbery/personal theft.

3.7. Data and Descriptive Statistics

3.7.1. The Offending, Crime and Justice Survey

The OCJS is an individual-level panel data set covering England and Wales in the period 2003-2006. It is similar in structure to the British Crime Survey. However,

[19] Foley's own interpretation of the results is that the permanent income hypothesis is violated and individuals suffer from self-control problems.

in addition to information regarding crime victimisation and individuals' socioeconomic position, the OCJS includes self-reports of offending. The OCJS was explicitly selected due to its richness regarding personal attitudes. This richness includes questions directly asking respondents for their views on the acceptability of committing crime. We interpret respondents' responses to these questions as a strong proxy for k.

The survey ran for four waves. The first wave, in 2003, consisted of a representative cross-sectional sample of 6,892 individuals aged 10-65 plus a boost sample of 3,187 individuals aged 10-25. Subsequently, the survey ran as a panel study with fresh sampling in every wave. In the waves after 2003, only those considered most likely to offend, i.e. those aged 10-25, were interviewed. Sampling was conducted at the household level using modified random sampling of addresses from the Postcode Address File.[20]

Since the theoretical model focuses on the relationship between the labour market and crime, it is important to focus on those individuals who are no longer required to be in full-time education. As such, analysis is performed only using data for respondents aged 17-25.[21] To address concerns regarding reverse causality, offending behaviour in period t is estimated using values of independent variables in period $t-1$.[22] Hence, only respondents completing interviews in two consecutive waves, a

[20]The random sampling was modified to ensure that in each of England and Wales's 43 PFAs at least 100 individuals were surveyed.

[21]17 is the lowest age when information is used to form independent variables in period $t-1$. For the study period, the minimum school leaving age was 16.

[22]As much of the OCJS data is inherently backward-looking, this approach is equivalent to observing independent variables at the start of a time period and offending behaviour during the corresponding time period.

"paired-transition", are included in the main analysis.[23] Thus, the main results use a sub-sample of 3,268 paired-transitions involving 2,004 individuals. This sub-sample is a highly unbalanced panel with just over half of respondents featuring in only one paired-transition.[24] Further detail on the structure of the unbalanced panel is given in Table 3.9.

With self-reported offending data, under-reporting is a concern. The OCJS was specifically designed to minimise under-reporting. First, data collection was performed by independent research companies rather than by the Home Office.[25] Second, to reassure respondents about the confidentiality of their data, respondents received letters on headed paper from the Home Office stating that the Home Office would not know the identity of those interviewed. Last, the interviews were designed to minimise interviewer influence. Responses concerning offending, drug use, alcohol use, health and risk factor questions were completed using computer assisted self-interviewing (CASI).

[23] A minimum amount of further data cleaning was undertaken. Three individuals were dropped for age discrepancies. Also, records involving partial interviews, i.e. interviews not reaching the offending questions, were dropped. Additionally, in 2004, data concerning personal "risk" factors was lost for some respondents. Respondents who were re-interviewed for this "risk" data several months after their original interview have had their 2004 data dropped. Following advice, those who reported ever having taken heroin were dropped due to re-contact and reliability problems. Lastly, the sub-sample is reduced by the requirement for respondents to have answered all questions relating to the dependent and independent variables.

[24] At present, the data is analysed without applying sampling weights. The only weights provided are for cross-sectional analysis and for fully-balanced panel analysis. The value of analysing the observations forming a fully balanced panel is probably limited. The sample of respondents aged 17-25 who are present in all four waves consists of only 305 individuals and 915 paired-transitions. Also, using weights designed to make the sample representative of the 10-25 population may well be inappropriate, given that the population of interest is those aged 17-25.

[25] The Home Office is the government department with responsibility for the police/law and order in the UK.

A small number of academic papers, and a range of Home Office reports, have made use of the OCJS. For example, Papadopoulos (2010) considers links between immigration and crime. However, none of this work specifically considers the relationship between the labour market and crime. Also, the Home Office reports[26] take a broader criminological view of the OCJS data. Hence, they use data for all those aged 10 and above rather than focusing on older, more economically active, age groups.

3.7.2. Descriptive Statistics

Crime variables and offending rates

The OCJS includes very detailed offending questions with over 20 different main offence categories being considered and a separate section covering "white-collar" crime. However, the low number of reports in many offence categories makes it necessary to aggregate the data into broader offence groups. Table 3.1 provides definitions and offending rates for each of the aggregate offence categories used. For now, other than selling stolen goods and credit card fraud, analysis of data from the "white-collar" crime section is left for future research.

As a comparison to the main paired-transition sample, another "Contemporary Sample" is reported. The only difference between this much larger sample, and the paired-transition sample is that in the former, data for both independent and dependent variables comes from the period t interview. Hence, individuals only need to be in one sampling wave to be included. All percentages for the descriptive statistics use the total number of observations, N, as their base unless stated otherwise.

[26] See, for example, Budd et al (2005), Wilson et al (2006) and Hales et al (2009).

Crime Variable	Definition	Paired-Transition Sample		Contemporary Sample	
		Offending Rate (%)	Total Number of Reports[1]	Offending Rate (%)	Total Number of Reports[1]
Theft	Includes: vehicle theft, burglary, robbery, shoplifting, theft from work and theft from school	10.50	343	9.58	541
Economic Crime	As for Theft but adding drug selling, credit card fraud and selling stolen goods	16.43	537	16.32	922
Economic Crime (ex. work and school theft)	As for Economic Crime but excluding theft from work and school	11.47	375	11.35	641

Note: "Paired-Transition Sample" refers to a sample where respondents answered all questions regarding independent variables in period t-1 and all questions regarding dependent variables in period t. "Contemporary Sample" refers to a sample where respondents answered all questions for both independent and dependent variables in period t. All percentages have N as their base.

[1] For "Paired-Transition Sample" as up to three paired transitions are covered each individual could make a maximum of three reports per crime category. For "Contemporary Sample" as an individual could be sampled in up to four waves they could make up to four reports per crime category.

Table 3.1: Definition of offence categories and offending rates by sample type.

The offending rates in the current sub-sample are in line with the offending rates reported in the Home Office reports using the OCJS. For example, for the 18-25 age group, Wilson et al (2006) state that 11% of individuals reported committing some form of theft and 5% sold drugs. Also, Budd et al (2005) take the 2003 data and compare it to data from the Home Office's Offenders Index.[27] The Offenders Index showed that 9% of males had a conviction by the age of 18-20. In the OCJS, the percentage of individuals, in the same age range, admitting some form of offence prior to interview was 63%. As discussed by Smith (2002), in the criminology literature self-reported offending rates are consistently found to be higher than those based on official data.

[27]This is a database holding conviction histories for 7 million individuals that covers all major crime types.

That self-reports of offending exceed the number of convictions is not surprising as only some crimes are detected/reported, the police only arrest a proportion of criminals and only a proportion of those arrested are actually convicted. Also, regarding reporting, some of the offences may occur within families. Others, such as workplace theft, involve a wide spectrum of behaviour. As such, not all reports of offending, had they been discovered, would have warranted a response from the criminal justice system. Within the sub-sample currently analysed, the total admissions of serious crimes, such as burglary and robbery, was very low (18 and 2 reports respectively).

The category Economic Crime (excluding work and school thefts) is included to overcome the following problem: if the unemployed do not have the opportunity to commit workplace theft, using a crime variable including workplace theft could bias downwards estimates for unemployment's impact on offending. Indeed, the offending rate for workplace theft of the unemployed was 3.51%, but for the employed it was 8.73%. However, for this bias to be serious, and for Economic Crime (ex. work and school theft) to be a better indicator of the unemployment-crime relationship, unemployed individuals must not substitute from workplace theft to other crimes. Whilst substitution probably does occur, it is plausible that workplaces may offer favourable opportunities for theft. The opportunities may be higher, and the risks lower, to take items from your employer's warehouse than to force entry into a house, or to steal and dispose of a car.

Respondent Characteristics

Table 3.2 reports the socioeconomic background of respondents. That period t values of independent variables are used in the contemporary sample explains why the mean age is approximately one year higher than in the paired-transition sample. This age difference may also explain some of the other differences in respondent characteristics between the two samples. All the offender/non-offender breakdowns refer to the paired-transition sample.

Statistic	**Non-Offenders (Economic Crime)**	**Offenders (Economic Crime)**	**Paired-Transition Sample**	**Contemporary Sample**
Personal/Household Characteristics				
% Male	42.99	64.43	46.51	46.04
Mean Age	19.98	19.46	19.89	21.03
(standard deviation)	(2.36)	(2.20)	(2.34)	(2.33)
% Non-white ethnicity	8.97	6.89	8.63	8.85
% A-Levels or above	55.36	50.65	54.59	63.96
% Live with parents	77.85	85.29	79.07	70.46
% Married or co-habiting	14.21	9.68	13.46	18.04
% Have biological children	10.95	5.59	10.07	12.80
% From home without 2 natural parents	26.14	26.44	26.19	26.90
% Religious (at 1st interview)	56.32	50.84	55.42	53.79
% Ever Sought mental health help (before 1st interview aged over 16)	19.33	27.75	20.72	22.76
% Victim of personal crime in past year	24.20	43.76	27.42	24.88
% Victim of household crime in past year	37.53	47.49	39.17	37.75
N	**2,731**	**537**	**3,268**	**5,650**
i	**1,768**	**428**	**2,004**	**3,105**

Note: "Paired-Transition Sample" refers to a sample where respondents answered all questions regarding independent variables in period t-1 and all questions regarding dependent variables in period t. "Contemporary Sample" refers to a sample where respondents answered all questions for both independent and dependent variables in period t. All percentages have N as their base and, other than for "Contemporary Sample", refer to period t-1. The breakdown by offending refers to the "Paired-Transition Sample" with the Offender/Non-Offender classification determined by responses to offending questions in period t.

Table 3.2: Respondents' personal and household characteristics.

Differences in the characteristics of offenders and non-offenders are immediately apparent. The most noticeable are the greater proportions of offenders who are males and victims of crime. The percentage of males is 21.44 percentage points higher for offenders than non-offenders and the percentage of offenders who were victims of personal crime is 19.56 percentage points higher.

The other significant feature of the data is that 79% of respondents lived with their parents. Whilst teenagers and young adults are those most likely to offend,[28] it is an open question whether such individuals are economically independent of their parents. Thus, those in the age group with the greatest proportion of offenders may supplement unemployment benefits with resources from other family members.

The introduction noted the benign labour market conditions during the OCJS's survey period. Table 3.3 confirms a low unemployment rate amongst those surveyed. That the unemployment rate for offenders is 1.26 percentage points lower than for non-offenders can be explained by the inclusion of workplace theft in the category Economic Crime.

[28] See Levitt (1999), Hales et al (2009), Budd et al (2005) and Wilson and Herrnstein (1985). There is consistent evidence that the proportion of the population who offend/get arrested declines with age. However, for continuing offenders, whether the frequency of offending declines with age is less clear (see Piquero et al (2007)). If older offenders are more persistant offenders, it would suggest individuals sort between legitimate and criminal activity over their lifetime.

Statistic	Non-Offenders (Economic Crime)	Offenders (Economic Crime)	Paired-Transition Sample	Contemporary Sample
Economic Variables				
% Unemployment rate[1]	8.46	7.20	8.26	8.02
% NEET rate[2]	13.22	10.24	12.73	13.99
Median household income category[3]	£20,000-24,999	£20,000-24,999	£20,000-24,999	£25,000-29,999
% Respondents answering "Don't know/Refused" to household income question	23.58	25.51	23.90	25.08
% Received free school meals as child[3]	19.10	22.08	19.59	20.47
% Interviewer reports rundown houses "Fairly/Very Common" in area[3]	9.23	10.75	9.48	10.26
N	**2,731**	**537**	**3,268**	**5,650**
i	**1,768**	**428**	**2,004**	**3,105**

Note: "Paired-Transition Sample" refers to a sample where respondents answered all questions regarding independent variables in period t-1 and all questions regarding dependent variables in period t. "Contemporary Sample" refers to a sample where respondents answered all questions for both independent and dependent variables in period t. All percentages have N as their base and, other than for "Contemporary Sample", refer to period t-1. The breakdown by offending refers to the "Paired-Transition Sample" with the Offender/Non-Offender classification determined by responses to offending questions in period t.

[1] The definition of unemployment is designed to match that of the Labour Force Survey. Based on the OCJS's employment status question the numerator is defined as those looking for employment/government training plus those waiting to take up paid employment. The denominator is formed from these two groups plus those in paid employment/self-employment and those doing unpaid work in a family business.

[2] NEET is an acronym for "Not in Education, Employment or Training". Based on the OCJS's employment status question NEET is defined as all respondents other than those going to school; going to college; in paid employment/self-employed; doing unpaid work for a family business; or on a government training scheme.

[3] The base for these percentages/calculating the median excludes those answering "Don't Know" or "Refused".

Table 3.3: Respondents' economic circumstances.

Statistic	Non-Offenders (Economic Crime)	Offenders (Economic Crime)	Paired-Transition Sample	Contemporary Sample
Risky/Negative Behaviours				
% Taken drugs in past year[1]	25.12	59.96	30.84	29.75
% Taken 'Class A' drugs in past year[1]	7.73	21.97	10.07	11.33
% Ever expelled (before 1st interview)	1.83	4.28	2.23	2.44
% Report friends in trouble with police[2]	18.37	40.90	22.09	19.46
% Parents ever in trouble with police (before 1st interview)[2]	8.20	11.76	8.79	8.51
% Parents ever in prison (before 1st interview)[2]	1.47	4.32	1.94	1.94
% Ever arrested (before 1st interview)	7.84	18.44	9.58	11.45
% Ever been to court (before 1st interview)	2.93	5.40	3.34	4.58
% Ever sentenced (before 1st interview)	2.05	3.72	2.33	3.29
% Ever sent to prison (before 1st interview)	0.15	0.74	0.24	0.39
N	**2,731**	**537**	**3,268**	**5,560**
i	**1,768**	**428**	**2,004**	**3,105**

Note: "Paired-Transition Sample" refers to a sample where respondents answered all questions regarding independent variables in period t-1 and all questions regarding dependent variables in period t. "Contemporary Sample" refers to a sample where respondents answered all questions for both independent and dependent variables in period t. All percentages have N as their base and, other than for "Contemporary Sample", refer to period t-1. The breakdown by offending refers to the "Paired-Transition Sample" with the Offender/Non-Offender classification determined by responses to offending questions in period t.

[1] Individuals who reported ever taking heroin were dropped from the sample due to re-contact problems. As such heroin use is not included in these statistics. 'Class A' drugs include cocaine. Cannabis has a lower (less serious) classification.

[2] The base for these percentages excludes those who answered "Don't Know" or "Refused".

Table 3.4: Respondents' engagement in risky or negative behaviours.

As one would expect, Table 3.4 shows that those who report offending are far more likely to report previous contact with the criminal justice system and engagement in risky behaviours during period $t-1$. In particular, the percentage of offenders taking drugs is 2.4 times (2.8 times for Class A drugs) the percentage of non-offenders. This, and the fact that 60% of offence reports came from individuals reporting prior drug use, is consistent with the theoretical model. It seems reasonable to suppose that those dependent on drugs have a particularly high marginal utility of additional consumption due to the high utility provided by obtaining an extra "fix". Considering NCC and NCC_E, if $u'(b)$ and $u'(w)$ are particularly high, drug users will require particularly high integrity, k, not to offend.

Some might argue that any link between Economic Crime and drug use simply reflects a "drugs culture" which inherently connects drug consumption and drug supply. However, if offending and non-offending are classified by Theft, a crime category that excludes selling drugs, the proportion of offenders taking drugs in period $t-1$ is still more than double that for non-offenders (60.6% versus 27.4%).

That only 0.2% of respondents admitted a spell in prison reflects two things. The first is the greater emphasis placed on community sentencing in the UK compared to, say, the US. Secondly, as the OCJS is a household survey, it excludes individuals currently in prison. Thus, the empirical results are probably most representative of those at an early stage in their criminal careers, "successful" criminals[29] or those who engage in relatively low-level offending.

Risk attitude and offending

The full question providing data for Figure 3.1 in the introduction was "Do you agree or disagree? I like taking risks in life". Table 3.10 in the Empirical Appendix records the responses to this question. The data for Figure 3.1 shows that responses of "Agree strongly" for "I like taking risks in life" were associated with offending rates between 3.6 and 4.7 times the offending rates of those responding "Disagree strongly". Figure 3.7 shows that it is also the case that offenders show a preference for taking risks.

[29] By "successful" criminals we mean those who have escaped conviction.

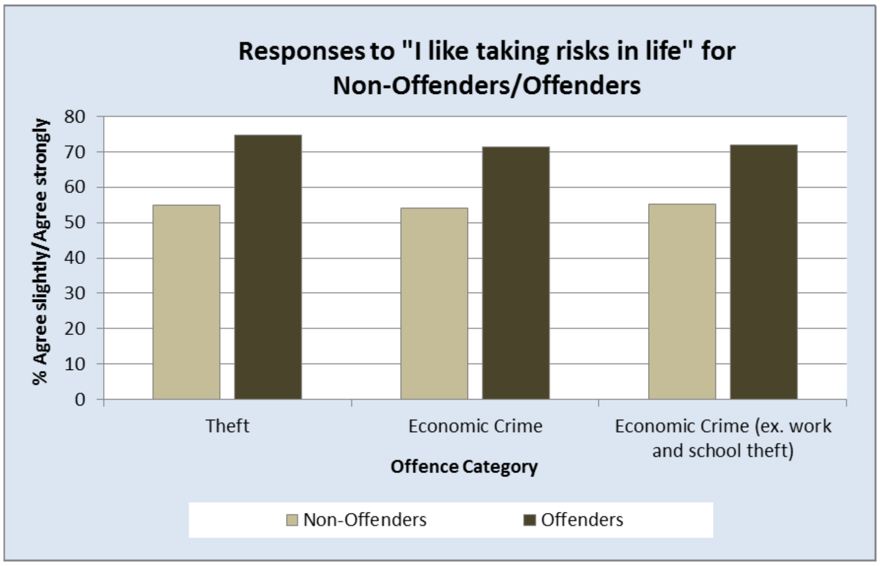

Figure 3.7: Attitude to risk at the end of period t by offending status during period t.

The data behind Figure 3.7 shows that, for each offence category, the percentage of offenders reporting "Agreed" or "Agreed strongly" to the risk taking statement was at least 16.8 percentage points higher than for non-offenders.

Integrity and offending

The theoretical model emphasises the central role that integrity, k, or the "psychic" cost of committing crime has on an agent's criminal decision. An original feature of the current chapter is access to data including clear proxies for k. The potential proxies are the responses to the following four questions:

"How much do you agree or disagree that....

- it is OK to steal something if you are very poor?

- it is OK to steal something from somebody rich who can afford to replace it?

- it is OK to steal something from a shop that makes a lot of money?

- it is sometimes OK to break the law?"

Respondents could answer each question on a five-point scale from "Strongly Agree" to "Strongly Disagree". If a respondent reported greater agreement with these statements, it is intuitive to interpret it as an indicator of their disutility from crime being lower. Table 3.5 highlights that responses were heavily skewed towards "Disagree" and "Strongly Disagree". Less than 0.5 percent of responses to the first three statements involved strong agreement.

Response	**OK to steal if poor**	**OK to steal from rich**	**OK to steal from shop**	**OK to sometimes break the law**
% Strongly agree	0.41	0.39	0.34	1.15
% Agree	3.26	1.13	0.94	18.28
% Neither agree nor disagree	8.94	3.42	3.61	21.79
% Disagree	39.89	44.02	43.08	36.34
% Disagree strongly	47.5	51.04	52.04	22.44
% Total[1]	**100.00**	**100.00**	**100.01**	**100.00**

Note: Attitudes to crime are held fixed at the value reported in the first interview to match the model. The base for the percentages is the total number of reports across all survey waves.

[1] Values that do not sum to 100% are due to rounding error.

Table 3.5: Responses regarding the acceptability of offending.

As is discussed in section 3.8, responses to the statement "it is sometimes OK to break the law" show the strongest relationship with offending. Hence, it is the responses to this question that have been used to form the integrity proxy. In the model, k is fixed through time and, to match this, the analysis fixes the responses to the crime attitude questions at the values given in a respondent's first interview. That offenders show disruptive/anti-social attitudes and behaviour from an early age has also been widely established in the criminology literature. For example, see Farrington (2002). Thus, when individuals enter our sub-sample at 17, their underlying views on offending are likely to be well established.

To reflect the slightly different spread of data for the "OK to steal" statements, Figure 3.8 shows offending rates by responses to the statement regarding theft when very poor. Figure 3.8 shows respondents reporting "Agree" have offending rates between 2.6 and 3 times higher than those reporting "Strongly disagree". For the breaking the law statement (Figure 3.2), the multiples are even higher being between 3.3 and 5.4. An exception to this pattern of increased offending when agreement with the statements increases is for respondents answering "Strongly agree". However, only a very small number of individuals, 9 in the case of the OK to steal if very poor statement, reported "Strongly agree". For the vast bulk of the data, a clear association exists between stronger agreement with crime being OK and subsequent offending.[30]

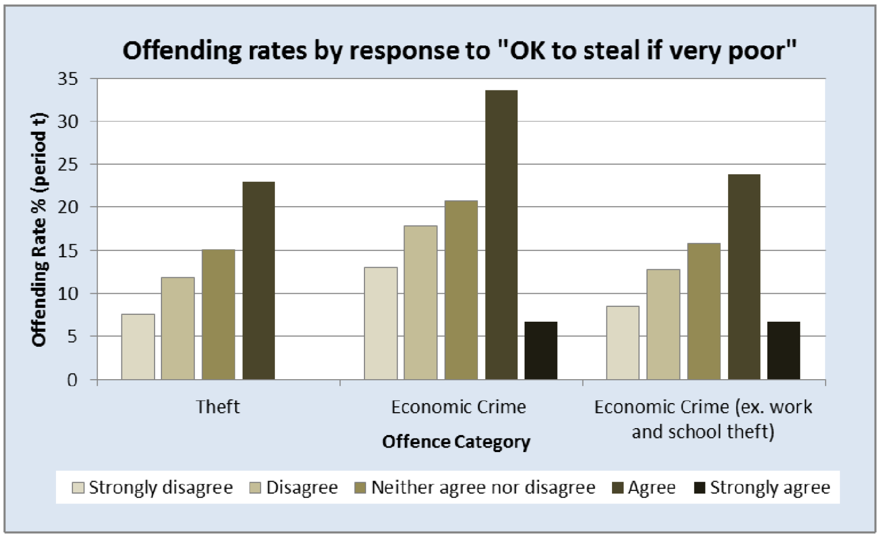

Figure 3.8: Offending rates in period t by attitude to stealing when poor, at first interview.

[30]The charts (not shown) for the other two stealing statements are very similar to the chart for the statement concerning stealing when very poor. The very low offending rate for those reporting strong agreement with the OK to steal statements seems related to religious belief. Of the 9 individuals who reported "Strongly agree" with it being OK to steal when very poor, 8 reported being a member of a religious group.

Liquidity constraints and offending

As a proxy for a binding liquidity constraint ($A = 0$) respondents' ratings of their household's financial position are used. Respondents were asked:

"Thinking of how your household is managing on your total income at the moment, would you say it was....

1. Managing quite well, able to save or spend on leisure,
2. Just getting by, unable to save if wanted to,
3. Getting into difficulties"

We interpret "Getting into difficulties" as a proxy for respondents approaching/having a binding liquidity constraint. Table 3.11 shows the proportion of responses in each category. Figure 3.9 shows the offending rates for those "Getting into difficulties" were between 4.9 and 9.3 percentage points higher than for those "Just getting by".

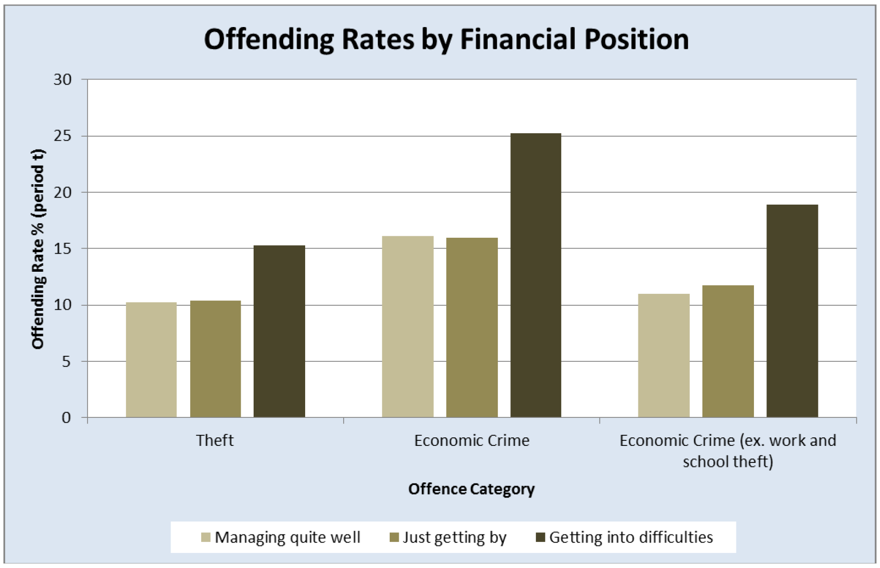

Figure 3.9: Offending rates in period t by financial position at the end of period t-1.

However, there were only 111 reports of "Getting into difficulties". So, whilst Figure 3.9 shows some support for the theoretical model's insight that liquidity constraints are linked to offending, it is unsurprising that the financial position dummies show only limited statistical significance in the econometric analysis.

Employment status and offending

Perhaps the most surprising feature of the OCJS data is the high level of offending reported by the employed. Indeed, for Theft and Economic Crime the offending rate for those in work is higher than for those out of work. This can be seen in Figure 3.10, below.

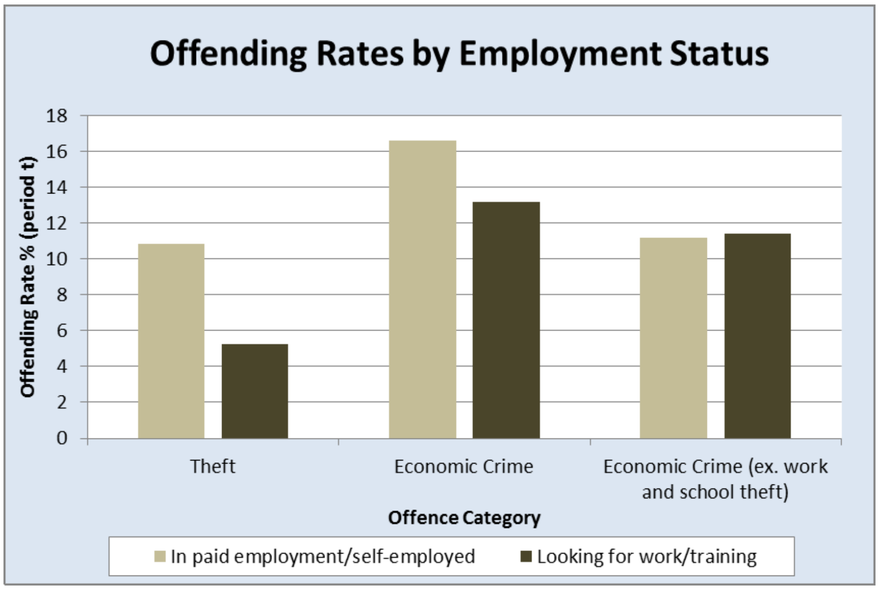

Figure 3.10: Offending rates in period t by employment status at the end of period t-1.

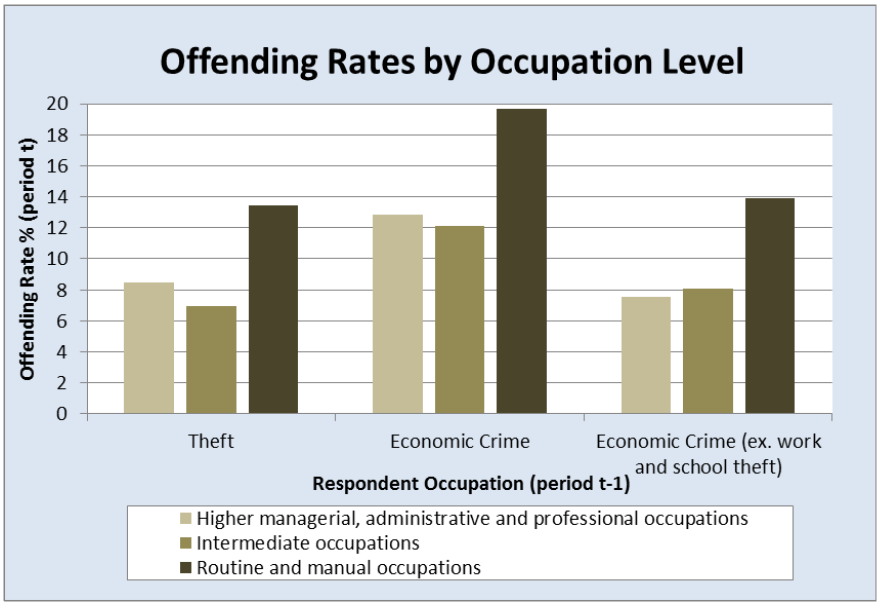

Figure 3.11: Offending rates in period t by occupation level at the end of period t-1.

Figure 3.10 show that, in the OCJS, the group reporting the highest offending rate is those in routine and manual occupations. It is the high offending rate amongst respondents in these low-level, presumably low-paid, occupations which drives the offending rates for Theft and Economic Crime to be higher for the employed than for those looking for work. This result is also explained by the high prevalence of workplace theft recorded. In 40.6% of interviews where the respondent reported committing Economic Crime, there was a report of stealing from work, and in 63.6% of interviews where Theft was admitted, this included stealing from work. Once one excludes workplace and school theft, the offending rate of those looking for work is over 3.3 percentage points higher than for those employed in intermediate or higher occupations.

As discussed earlier in this section, these results may reflect employed individuals having more opportunities for criminal activity. Beyond this, the survey period, 2003-2006, was a period of benign economic conditions. This fact is central to understanding these results. The unemployment rate for 18-24 year olds during 2003-2006 was in the range 9.9%-12.6%. This compares to 19.7% for the year ending June 2012.[31] In these favourable conditions, it appears even "criminally inclined" individuals could find employment. Also, note that for those under 20, being in employment, rather than in full-time education, may indicate low future earnings. The model suggests that for such individuals the opportunity cost of jail is probably low.

Some agents will switch between committing crime whilst unemployed and not committing crime whilst employed. Again, the benign economic conditions when the OCJS was conducted probably meant that the group of unemployed "unfortunates" was small.

Another possible reason why unemployed young adults did not report higher offending rates is that they lived with their parents. Almost 80% of respondents in the OCJS lived with their parents. For these individuals the difference in utility when employed and when unemployed may have been low. Their unemployment benefits may have been supplemented with other household resources; i.e. they may have used the "bank of mum and dad".

The full details of respondents' employment statuses are provided in Table 3.12. Given the high proportion of respondents living with their parents, Table 3.12 also

[31]These figures are based on Labour Force Survey data.

includes information regarding the Household Reference Person's (HRP's)[32] employment status.

Table 3.12 provides further context for Figure 3.11. Table 3.12 shows that a greater proportion of offenders than non-offenders previously reported activities which could represent "disguised" unemployment. For example, a higher percentage of offenders previously reported "intending to look for work but prevented by temporary sickness or injury".

Overall, Table 3.12 is consistent with the difference between $u'(b)$ and $u'(w)$ being low. In Table 3.12, over half of workers report being in routine and manual occupations. Not only are these jobs likely to be low paid, but they probably also have poor non-pecuniary characteristics.[33] Also, table 3.12 shows evidence regarding the capacity of HRPs to provide resource transfers to unemployed household members. In just over 80% of the paired-transitions, the HRP was in paid employment/self-employment. Additionally, in 36% of paired-transitions, the HRP was employed in a presumably well-paid, higher managerial, administrative or professional occupation.

[32]The HRP is identified as the person who owns/rents the household's accommodation. If accommodation is held in joint names, the individual with the highest income becomes the HRP. If individuals also share a common income level, then the HRP is the oldest individual in the household.

[33]If the value of being employed is low the opportunity cost of being in jail, $V^J(A) - V^E(A)$, is also reduced.

3.8. Econometric Analysis

3.8.1. Econometric Method

In all of the estimations, offending is modelled as a binary choice with the offending behaviour of individual i in period t being represented by O_{it}.[34] O_{it} takes a value of 1 when offending is reported and a value of 0 when no offending is reported. The probability of each outcome occurring is:

$$O_{it} = \left\{ \begin{array}{cc} 1 & \text{with probability } p_{it} \\ 0 & \text{with probability } 1 - p_{it} \end{array} \right\}$$

The aim is to model p_{it} as a function of time-invariant and time-varying independent variables. The baseline model is a straightforward probit estimation.[35] Beyond this, a fixed-effects logit model, a biprobit model with partial observability and a complementary log-log model have also been estimated.

As mentioned in Section 3.7, the time-varying independent variables are lagged by one period to reduce the risk of two-way causation biasing the results. Whilst the offending questions refer to the 12 months prior to interview, many of the independent variable questions relate to the respondent's position at the point of interview. Using dependent and independent variables from the same interview wave creates the following problem. Suppose someone at the end of period t reports offending during period

[34]A count data model is not used due to the low proportion of individuals who offend.
[35]The two baseline probit specifications, specifications 1 and 2, have also been estimated using the logit link function. The differences in the values of the maximised log-likelihood functions are always less than 1%. As such, there is no advantage in using a logit model over the probit model.

t and that, at present, they are unemployed. The question then arises of whether the respondent committed the offence after becoming unemployed, or, whether the offence led to the individual being sacked, implying that offending caused the unemployment? Taking the first lag of the unemployment indicator, removes this issue. Hence, p_{it} is modelled as:

$$p_{it} \equiv P(O_{it} = 1|\mathbf{x}_{it-1}, \mathbf{y}_i) = F(\mathbf{x}'_{it-1}\boldsymbol{\beta} + \mathbf{y}'_i\boldsymbol{\gamma})$$

where $\mathbf{x}_{it-1}$ is a vector of independent variables which vary by individual and time, $\mathbf{y}_i$ is a vector of time-invariant independent variables, and $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are vectors of coefficients to be determined.

In the probit model, $F(.)$ is specified as the Normal cumulative distribution function. The transformation $F(.)$ ensures the estimated value of p_{it} lies between zero and one.

Using full panel data methods on the paired-transition sample does not appear feasible. Table 3.9 shows that 52% of respondents took part in only a single paired-transition. Instead, a pooled cross-section approach is used. Estimation is performed using maximum likelihood techniques. For a sample of N paired-transitions, the log-likelihood function which the estimators $\widehat{\boldsymbol{\beta}}$ and $\widehat{\boldsymbol{\gamma}}$ maximise is:

$$Q(\boldsymbol{\beta}, \boldsymbol{\gamma}) = \sum_{i=1}^{N}\sum_{t=2}^{T}\left[O_{it}\ln F(\mathbf{x}'_{it-1}\boldsymbol{\beta} + \mathbf{y}'_i\boldsymbol{\gamma}) + (1 - O_{it})\ln\left(1 - F(\mathbf{x}'_{it-1}\boldsymbol{\beta} + \mathbf{y}'_i\boldsymbol{\gamma})\right)\right]$$

Recognising that the error terms for each individual i are almost certainly correlated through time, a cluster robust estimate for the variance-covariance matrix is used.

Each individual, i, is treated as a separate cluster. However, independence of the error terms between individuals is still assumed.

All the independent variables are described in Table 3.13. Separate estimations were performed for each crime category identified in Table 3.1. A variable representing attitude to risk is not included in the estimations, as the relationship between attitude to risk and offending emerges from the model endogenously. Including a regressor, which the model implies is endogenous, is unattractive as it leads to the maximum likelihood estimators being inconsistent.

Two versions of the baseline probit model were run. Compared to specification 1, specification 2 includes an extra variable recording whether individuals reported offending prior to their first interview. In the context of explaining why individuals offend, there is value in running the estimations without this prior offending variable. It seems natural for this prior offending variable to "swamp" the other independent variables' explanatory power without providing much insight about why individuals offend. However, the prior offending variable can also be interpreted as a further proxy for integrity. It indicates that previously a respondent's value of k was sufficiently low for it to lie below the NCC/NCC_E. Yet, since in reality wages, benefits and time spent in jail may vary through time, causing the NCC/NCC_E to also shift through time, there is perhaps a better interpretation. This prior offending variable is best used to identify all the unobservable characteristics that make an individual likely to commit crime. In this context, specification 1 identifies factors associated with offending, whilst specification 2 indicates whether these factors are robust to including a control for individuals' unobservable characteristics.

3.8.2. Results

Table 3.6 reports the average marginal effects for the baseline probit using specifications 1 and 2. Along with the variables relating to financial position, employment status and integrity, other variables are reported which are consistently significant at the 1% level, or which have particular relevance to offending. Apart from "Age", all the independent variables are binary variables or categorical variables broken down into dummies. The values not in parentheses, therefore, report the average discrete change in the probability of offending, p_{it}, when a variable shifts from its "Null" position (shown in Table 3.13) to the position stated. The marginal effects for these binary/dummy variables are calculated using finite-difference methods. All statements regarding statistical significance relate to Wald tests.

Considering specification 1 first, the association between respondents' attitude to breaking the law and subsequent offending is statistically significant and in the expected direction. For all three crime categories, as one moves from "Agree" towards disagreement, the average marginal effects are negative and, in all but two cases, are statistically significant at the 1% level.[36]

[36] The other two cases are statistically significant at the 5% level.

Independent Variable	Specification 1 - Baseline Probit: Theft	Specification 1 - Baseline Probit: Economic Crime	Specification 1 - Baseline Probit: Economic Crime (ex. work and school theft)	Specification 2 - Prior Offending Control: Theft	Specification 2 - Prior Offending Control: Economic Crime	Specification 2 - Prior Offending Control: Economic Crime (ex. work and school theft)
Household just getting by on income	0.017	0.019	0.023*	0.015	0.017	0.022*
	(0.013)	(0.015)	(0.013)	(0.013)	(0.015)	(0.013)
Household getting into difficulties on income	0.048	0.074*	0.047	0.034	0.061	0.038
	(0.035)	(0.041)	(0.032)	(0.031)	(0.038)	(0.031)
Respondent employment status: intermediate	-0.005	-0.011	-0.009	-0.013	-0.017	-0.009
	(0.023)	(0.028)	(0.026)	(0.022)	(0.028)	(0.025)
Respondent employment status: routine and manual occupations	0.030	0.027	0.013	0.026	0.023	0.013
	(0.021)	(0.026)	(0.023)	(0.020)	(0.026)	(0.022)
Respondent employment status: looking for paid work/training	-0.046*	-0.049	-0.028	-0.042	-0.044	-0.026
	(0.026)	(0.035)	(0.030)	(0.027)	(0.035)	(0.030)
Strongly agree: sometimes OK to break the law (1st interview)	-0.051	-0.076*	-0.066*	-0.020	-0.049	-0.040
	(0.038)	(0.045)	(0.037)	(0.042)	(0.049)	(0.040)
Neither agree/disagree: sometimes OK to break the law (1st interview)	-0.039**	-0.044**	-0.045***	-0.021	-0.030	-0.030*
	(0.019)	(0.021)	(0.017)	(0.016)	(0.019)	(0.015)
Disagree: sometimes OK to break the law (1st interview)	-0.053***	-0.061***	-0.046***	-0.024	-0.031*	-0.023
	(0.017)	(0.020)	(0.016)	(0.016)	(0.018)	(0.015)
Strongly disagree: sometimes OK to break the law (1st interview)	-0.099***	-0.099***	-0.056***	-0.066***	-0.062***	-0.030*
	(0.017)	(0.020)	(0.017)	(0.016)	(0.020)	(0.016)
Taken drugs in past year (not 'Class A')	0.087***	0.134***	0.107***	0.058***	0.099***	0.079***
	(0.015)	(0.018)	(0.015)	(0.014)	(0.017)	(0.015)
Taken 'Class A' drugs in past year	0.076***	0.162***	0.153***	0.039**	0.112***	0.110***
	(0.021)	(0.027)	(0.024)	(0.017)	(0.024)	(0.022)
Friends in trouble with police in past year	0.059***	0.078***	0.052***	0.046***	0.064***	0.039***
	(0.013)	(0.016)	(0.013)	(0.012)	(0.015)	(0.012)
Victim of crime in past year	0.026**	0.026**	0.021**	0.018*	0.017	0.015
	(0.011)	(0.013)	(0.011)	(0.010)	(0.012)	(0.011)
Ever expelled (before 1st interview)	0.012	0.103**	0.090**	0.011	0.103**	0.080*
	(0.037)	(0.047)	(0.046)	(0.035)	(0.045)	(0.044)
Ever arrested (before 1st interview)	0.016	0.054**	0.052**	0.003	0.035	0.038*
	(0.020)	(0.024)	(0.022)	(0.018)	(0.023)	(0.021)
Ever sent to prison (before 1st interview)	0.084	0.230	0.191	0.039	0.167	0.127
	(0.111)	(0.194)	(0.181)	(0.087)	(0.167)	(0.151)
Ever committed economic crime (before 1st interview)[1]	-	-	-	0.114***	0.122***	0.093***
				(0.013)	(0.016)	(0.015)
Male	0.035***	0.063***	0.053***	0.026**	0.053***	0.047***
	(0.012)	(0.014)	(0.012)	(0.011)	(0.014)	(0.012)
Age	-0.003	-0.006	-0.007**	-0.001	-0.004	-0.007**
	(0.003)	(0.004)	(0.003)	(0.003)	(0.004)	(0.003)
Ever sought help for mental health problems (before 1st interview over age 16)	0.044***	0.072***	0.055***	0.042***	0.068***	0.052***
	(0.016)	(0.019)	(0.016)	(0.015)	(0.018)	(0.015)
PFA: Derbyshire	-0.088**	-0.124***	-0.110***	-0.096***	-0.129***	-0.104***
	(0.036)	(0.041)	(0.032)	(0.033)	(0.039)	(0.032)

(continued on following page)

	Theft	Economic Crime	Economic Crime (ex. work and school theft)	Theft	Economic Crime	Economic Crime (ex. work and school theft)
PFA: Devon & Cornwall	-0.068*	-0.109***	-0.095***	-0.074**	-0.114***	-0.090***
	(0.036)	(0.042)	(0.035)	(0.036)	(0.040)	(0.033)
PFA: Essex	-0.109***	-0.157***	-0.131***	-0.117***	-0.159***	-0.123***
	(0.035)	(0.039)	(0.026)	(0.034)	(0.039)	(0.026)
PFA: North Yorkshire	-0.116***	-0.162***	-0.108***	-0.121***	-0.164***	-0.089**
	(0.032)	(0.040)	(0.035)	(0.031)	(0.039)	(0.041)
Sweep 3	-0.041**	-0.056**	-0.031	-0.044**	-0.053**	-0.030*
	(0.019)	(0.022)	(0.019)	(0.018)	(0.022)	(0.018)
N	**3,268**	**3,268**	**3,268**	**3,268**	**3,268**	**3,268**
i	**2,004**	**2,004**	**2,004**	**2,004**	**2,004**	**2,004**
Log likelihood	**-889.51**	**-1,166.84**	**-899.66**	**-839.44**	**-1,127.88**	**-874.45**
Median predicted probability of offending report	**0.062**	**0.106**	**0.059**	**0.047**	**0.095**	**0.054**
p-value for joint test of managing on income H_0: =0	**0.327**	**0.239**	**0.032**	**0.484**	**0.316**	**0.024**
p-value for joint test of employment status H_0: =0	**0.048**	**0.166**	**0.536**	**0.099**	**0.264**	**0.639**
p-value for joint test of 'OK to break the law' H_0: =0	**0.000**	**0.000**	**0.011**	**0.001**	**0.035**	**0.274**

Notes: Cluster robust standard errors are given in parentheses. Significance levels: * 10% significance, ** 5% significance and *** 1% significance. The p-values reported test whether the marginal effects are jointly different from zero for the set of independent variables stated. Specification 2 is the baseline probit estimation with an extra dummy variable indicating whether a respondent reports offending prior to their first interview. Independent variables which were frequently significant at the 5% level or above but not shown here for brevity are: Household income: £35,000-£44,999 (positive); Drinks alcohol 1-3 times a month (positive); Household size:1 (negative); PFA: Dyfed Powys (negative); PFA: Hampshire (negative); PFA: West Mercia (negative); PFA: Wiltshire (negative); Sports club/gym member (positive); and Not 100% truthful re: crime questions (positive). Many other independent variables were also significant in individual regressions at the 10% level or above.

[1] This variable varies by dependent variable. If the dependent variable is "Theft" then this variable is whether the respondent has ever committed "Theft" before their first interview.

Table 3.6: Average marginal effects for the baseline probits using specifications 1 and 2.

The average drop in offending probability also becomes larger as the level of disagreement with "it is sometimes OK to break the law" becomes stronger. For example, in the cases of Theft and Economic Crime, whilst moving from "Agree" to "Neither agree/disagree" is associated with an average fall in p_{it} of around 4 percentage points, moving from "Agree" to "Strongly disagree" is associated with a 9.9 percentage point drop. Also, Wald tests reject the null hypothesis that all the integrity proxy dummies are equal to zero. Whilst it is anomalous that the shift from "Agree" to "Strongly Agree" for "it is sometimes OK to break the law" is associated with a reduction in p_{it}, this result is only weakly significant.[37]

[37] Given the small sample, relationships significant only at the 10% level are likely to be particularly weak.

Not only is our proxy for k highly statistically significant, but the magnitudes of the average marginal effects for a shift from "Agree" to "Strongly disagree" also appear empirically relevant. For all three classifications of crime, the reduction in p_{it} is of a greater magnitude than the increases in p_{it} associated with reporting friends in trouble with the police, being male, being a victim of crime or having previously sought help for mental health problems. However, apart from for Theft, taking drugs has a noticeably greater impact on p_{it} than the integrity proxy. For the two Economic Crime variables, drug taking is associated with an increase in p_{it} of between 10.7 and 16.2 percentage points. Nevertheless, specification 1 provides strong support for the importance of integrity in individuals' criminal decisions.

The only other dummies that have statistically significant average marginal effects of a similarly large magnitude to drug taking, are those for some of the PFA fixed effects. Also, in specification 1 there are no PFAs that show an increase in p_{it} (compared to the Metropolitan PFA) significant at the 5% level. The PFAs which show large and statistically significant drops in p_{it} are all considerably more rural than London. However, as there were 41 PFA dummies, it is surprising that more did not have statistically significant marginal effects.[38]

In contrast to the integrity proxy, the associations of financial position and employment status with offending are both weak. Only rarely are the average marginal effects statistically significant at the 10% level.

[38]Beyond picking up rural-urban differences, the PFA fixed effects should also capture differences in policing methods/resources and local labour market/economic characteristics.

Looking in detail at the financial position dummies, the magnitude of the average increase in p_{it} when reporting "getting into difficulties" is reasonably large, being 7.4 percentage points for Economic Crime.[39] In addition, for Economic Crime (ex. work and school theft) the average marginal effects for the financial position variable, when tested jointly are significantly different from zero at the 5% level. This result remains true in specification 2. As previously suggested, the lack of statistical significance for "getting into difficulties" may be due to the small number of individuals in this category. Hence, overall, the data provides tentative signs that financial position may play a role in determining offending.

The number of unemployed individuals within the sample is also small. Nevertheless, for all three crime categories, the negative sign for the average marginal effect of looking for work/training is the opposite to our initial expectations. However, these negative marginal effects are only statistically significant for Theft, and here the significance is only at the 10% level.[40] Specification 1 has also been run using wider categories for unemployment and replacing the employment status of the respondent with that of the household head (HRP). Neither approach led to the average marginal effect becoming positive, although, when the widest definition of unemployment was used, the magnitude of the negative average marginal effect was reduced to 1.3-1.5 percentage points.[41]

[39] Also, the raw co-efficient for "getting into difficulties" in the probit estimation for Economic Crime using specification 1 is positive and significant at the 5% level.

[40] When the categories of those looking for work and those waiting to take up employment already obtained are combined to match the Labour Force Survey's definition of unemployment, the average marginal effect for Theft is no longer statistically significant.

[41] The widest definition of unemployment included those responses that might cover "disguised" unemployment. Beyond waiting to take up paid employment already obtained, the additional responses included were: being on a government training scheme, intending to look for work but prevented from doing so by sickness, and doing something else.

As expected, the average marginal effect in specification 1 with the largest magnitude is for spending time in prison prior to the respondent's first interview. However, as only 4 individuals reported spending time in prison, this average marginal effect is not significant. The average marginal effects for the Economic Crime variables of being expelled or being arrested, although of a lower magnitude, are both significant at the 5% level.

Given that the criminology literature's identifies a declining age-crime profile after the late teenage years, one slightly surprising finding is that Age only has a statistically significant negative relationship with Economic Crime (ex. work and school theft). There are a number of explanations for this. Firstly, the age variation being considered, 17 to 25, is relatively small. Secondly, there are other age-related variables, such as highest educational qualification obtained, living with parents and having a child which are included in the regressions. Lastly, and perhaps most importantly, as young adults age, they move out of education into employment. Using OCJS data, Hales et al (2009) note that in contrast to other forms of theft, the rate of workplace theft continues rising until age 20 (shoplifting peaks at around 14 to 15), and then falls only relatively slowly. This last reason can explain the difference in the significance of Age between the crime categories, i.e. only after workplace theft is excluded is a significant negative relationship found.

Moving to specification 2, which includes the prior offending control, many variables experience a loss of significance compared to specification 1. In particular, there are marked drops in the number of average marginal effects for the integrity proxy, which are highly statistically significant. Nevertheless, a statistically significant relationship with offending does still exist for large shifts in respondents' attitude to crime. Also, for Theft and Economic Crime, Wald tests still reject the joint hypothesis that all the integrity dummies are equal to zero.

The average marginal effects of admitting offending prior to first interview are always significant at the 1% level. The magnitudes of these average marginal effects are also large. Admitting an offence prior to first interview is associated with a 9.3 to 12.2 percentage point increase in p_{it}. The general loss of significance for the integrity proxy suggests, unsurprisingly, that integrity and prior offending are highly correlated.

Whilst predicting the probability of offending for different individuals is not this study's purpose, it is worth considering how the magnitudes of the average marginal effects compare to the predicted probabilities of offending, $\widehat{p}_{it}$. Table 3.14 shows the predicted values of p_{it} are heavily skewed towards zero, i.e. not offending. In all specifications, over 48% of the predictions are for $\widehat{p}_{it} < 0.1$.[42] Whilst Table 3.14 and the median values of $\widehat{p}_{it}$ in Table 3.6 reinforce the empirically relevant magnitude of the average marginal effects, a note of caution should be struck. These average marginal effects are just that: averages. To gain a greater understanding of how the marginal effects vary by respondent, six hypothetical individuals have been considered. The characteristics of these individuals are described in Table 3.15. For each of these hypothetical individuals, marginal effects have been calculated using their

[42]The highest is 67%.

characteristics as representative values. These marginal effects are reported in Table 3.7.

Independent Variable	Successful Graduate	Single mother	A-Level Student	Family man	Rogue	Average Joe
Household just getting by on income	0.010	0.018	0.032	0.008	0.037	0.031
	(0.009)	(0.015)	(0.025)	(0.009)	(0.028)	(0.025)
Household getting into difficulties on income	0.037	0.062	0.114*	0.027	0.130*	0.109*
	(0.023)	(0.048)	(0.059)	(0.027)	(0.067)	(0.061)
Respondent employment status: intermediate occupation	-0.006	-0.011	-0.020	-0.005	-0.023	-0.019
	(0.017)	(0.028)	(0.050)	(0.012)	(0.057)	(0.048)
Respondent employment status: routine and manual occupations	0.014	0.024	0.043	0.010	0.049	0.041
	(0.014)	(0.025)	(0.044)	(0.013)	(0.050)	(0.043)
Respondent employment status: looking for paid work/training	-0.031	-0.052	-0.095	-0.022	-0.108	-0.091
	(0.027)	(0.046)	(0.071)	(0.026)	(0.085)	(0.071)
Strongly agree: sometimes OK to break the law (1st interview)	-0.039	-0.066	-0.121	-0.028	-0.138	-0.116
	(0.030)	(0.052)	(0.084)	(0.031)	(0.095)	(0.084)
Neither agree/disagree: sometimes OK to break the law (1st interview)	-0.021	-0.036	-0.065**	-0.015	-0.074**	-0.062*
	(0.013)	(0.022)	(0.033)	(0.015)	(0.036)	(0.033)
Disagree: sometimes OK to break the law (1st interview)	-0.031**	-0.052*	-0.095***	-0.022	-0.108***	-0.091**
	(0.014)	(0.030)	(0.034)	(0.021)	(0.035)	(0.037)
Strongly disagree: sometimes OK to break the law (1st interview)	-0.054**	-0.091**	-0.167***	-0.039	-0.190***	-0.160***
	(0.024)	(0.046)	(0.045)	(0.035)	(0.046)	(0.052)
Taken drugs in past year (not 'Class A')	0.067**	0.112**	0.205***	0.048	0.234***	0.196***
	(0.026)	(0.053)	(0.048)	(0.042)	(0.043)	(0.053)
Taken 'Class A' drugs in past year	0.077**	0.130**	0.238***	0.056	0.271***	0.228***
	(0.031)	(0.061)	(0.054)	(0.049)	(0.050)	(0.065)
Friends in trouble with police in past year	0.039**	0.066*	0.121***	0.028	0.138***	0.115***
	(0.017)	(0.035)	(0.034)	(0.025)	(0.033)	(0.040)
Victim of crime in past year	0.014*	0.024	0.044*	0.010	0.050*	0.042**
	(0.008)	(0.015)	(0.023)	(0.010)	(0.026)	(0.021)
Ever expelled (before 1st interview)	0.049*	0.082*	0.151**	0.035	0.172**	0.144**
	(0.028)	(0.047)	(0.064)	(0.034)	(0.068)	(0.069)
Ever arrested (before 1st interview)	0.027*	0.046*	0.084**	0.020	0.096**	0.080**
	(0.016)	(0.027)	(0.036)	(0.019)	(0.042)	(0.038)
Ever sent to prison (before 1st interview)	0.096	0.162	0.297	0.070	0.338	0.283
	(0.077)	(0.138)	(0.217)	(0.077)	(0.248)	(0.208)
Male	0.034**	0.058*	0.106***	0.025	0.121***	0.101***
	(0.015)	(0.031)	(0.032)	(0.023)	(0.032)	(0.034)
Age	-0.003	-0.005	-0.010	-0.002	-0.011	-0.010
	(0.002)	(0.005)	(0.006)	(0.002)	(0.007)	(0.006)
Ever sought help for mental health problems (before 1st interview over age 16)	0.037**	0.062*	0.114***	0.027	0.130***	0.109***
	(0.016)	(0.033)	(0.033)	(0.024)	(0.037)	(0.037)

(continued on following page)

	Successful Graduate	Single mother	A-Level Student	Family man	Rogue	Average Joe
PFA: Derbyshire	-0.077*	-0.129	-0.236**	-0.055	-0.269**	-0.226**
	(0.046)	(0.080)	(0.102)	(0.053)	(0.112)	(0.107)
PFA: Devon & Cornwall	-0.064	-0.107	-0.196**	-0.046	-0.224**	-0.187*
	(0.040)	(0.070)	(0.092)	(0.045)	(0.103)	(0.096)
PFA: Essex	-0.111*	-0.187*	-0.342**	-0.080	-0.390***	-0.327**
	(0.063)	(0.106)	(0.135)	(0.075)	(0.149)	(0.146)
PFA: North Yorkshire	-0.119*	-0.200*	-0.366**	-0.086	-0.416**	-0.349**
	(0.068)	(0.120)	(0.153)	(0.079)	(0.169)	(0.162)
Sweep 3	-0.031*	-0.053*	-0.097**	-0.023	-0.110**	-0.092**
	(0.017)	(0.032)	(0.043)	(0.021)	(0.049)	(0.044)
N	**3,268**	**3,268**	**3,268**	**3,268**	**3,268**	**3,268**
i	**2,004**	**2,004**	**2,004**	**2,004**	**2,004**	**2,004**
Predicted probability of reporting Economic Crime	**0.054**	**0.107**	**0.195**	**0.039**	**0.607**	**0.257**
p-value for joint test of managing on income H_0: =0	**0.382**	**0.570**	**0.224**	**0.795**	**0.215**	**0.302**
p-value for joint test of employment status H_0: =0	**0.765**	**0.809**	**0.340**	**0.990**	**0.383**	**0.585**
p-value for joint test of 'OK to break the law' H_0: =0	**0.241**	**0.403**	**0.007**	**0.868**	**0.001**	**0.051**

Notes: Cluster robust standard errors are given in parentheses. Significance levels: * 10% significance, ** 5% significance and *** 1% significance. The p-values reported test whether the marginal effects are jointly different from zero for the set of independent variables stated.

Table 3.7: Marginal effects on the probability of Economic Crime, for six hypothetical individuals.

Table 3.7 shows the marginal effects' magnitudes, as well as their significance, varies considerably between the hypothetical individuals. The general pattern is for the hypothetical individuals with higher values of $\widehat{p}_{it}$ to have marginal effects of a higher magnitude and greater statistical significance. The clearest illustration of this is the contrast between the marginal effects for a hypothetical "Family Man" and a hypothetical "Rogue". For the hypothetical, "Family Man", none of the reported variables are statistically significant, whereas 15 of the reported variables are statistically significant at the 5% or 1% levels for the hypothetical "Rogue". This exercise suggests that for a typical non-offender to switch to being an offender, a range of factors must change.

Overall, these probit estimations provide strong support for the notion that an individual's attitude to crime, which we interpret as a clear proxy for k, is related to subsequent offending. Even though the statistical significance of attitude to crime drops once a control for prior offending is included, this control itself could be taken as another proxy for integrity. Also, the prior offending variable demonstrates the role individuals' unobservable characteristics play in determining their offending decisions.

There is also some tentative evidence that those individuals experiencing financial difficulties are more likely to offend. Where the results and data are more surprising, is in the lack of relationship between employment status and offending. This appears to be driven, in part, by the prevalence of workplace theft reported in the OCJS, which suggests most crime was being committed by the "criminally inclined". It is also plausible that the benign economic conditions during the survey period meant that few individuals with the characteristics of an "unfortunate" were actually out of work.

Whilst it is difficult to make direct comparisons between studies, due to the different samples and estimation techniques used, the work of Hales et al (2009), which also uses the OCJS, suggests a similar pattern of significance across the variables common to both studies. As in the present study, showing approval for criminal activities, being a victim of crime, being excluded from school, having friends in trouble with the police and being male all increased p_{it}.

3.9. Robustness

A wide range of alternative specifications have been estimated to ensure the robustness of the results reported in Table 3.6. Much of this testing involved running modified versions of specification 1. The exceptions to this were attempts to control for zero inflation by estimating a fixed effects logit model and a bivariate probit model with partial observability. More detail regarding these alternative estimation approaches is provided in the Empirical Appendix.

Robustness of the baseline specification

Due to the lack of significance of age in specifications 1 and 2, these specifications were re-run with terms for age squared and age cubed added. In neither specification were these extra variables significant. Additionally, to test for possible misspecification, RESET tests were performed. The RESET test includes squared and cubed terms of the fitted values of the index, $\mathbf{x}'_{it-1}\widehat{\boldsymbol{\beta}} + \mathbf{y}'_i\widehat{\boldsymbol{\gamma}}$, as additional regressors. If the terms are significant, it suggests the model is potentially mis-specified or, for the probit model, the error terms are non-Normal. The results in Table 3.8 suggest that the Theft regressions could be mis-specified. However, as predicting offence probabilities is not the focus of the paper, the importance of this result should not be overstated.

Given the large number of dummy variables in the regressions, tests were also performed to check for multicollinearity. In no case was multicollinearity identified.

Regression Term	Specification 1 - Baseline Probit			Specification 2 - Prior Offending Control		
	Theft	Economic Crime	Economic Crime (ex. work and school theft)	Theft	Economic Crime	Economic Crime (ex. work and school theft)
Index squared	0.129	0.293	0.186	0.001	0.071	0.501
Index cubed	0.573	0.545	0.236	0.008	0.095	0.553
Joint test squared and cubed terms	0.003	0.284	0.414	0.000	0.194	0.790

Notes: The figures reported are p-values for Wald tests of the null hypothesis that the co-efficient for the variable stated is equal to zero.

Table 3.8: P-values from RESET tests.

The baseline probit model has also been run for two additional specifications. The average marginal effects for these specifications are reported in Table 3.16. In specification 3, dummy variables representing all four crime attitude questions are included. Including this additional information does not alter the overall pattern of significance. It also shows that only dummies for the "sometimes OK to break the law" statement have average marginal effects consistently significant at the 5% level. This supports the choice of the "sometimes OK to break the law" statement as the integrity proxy used in specifications 1 and 2.

Since the distribution of responses is heavily skewed towards not offending, $O_{it} = 0$, it is sensible to assess whether the assumed symmetry of the error terms in the probit model is reasonable. To evaluate this assumption, specification 1 was also run using a complementary log-log link function. The complementary log-log model allows the error terms to be asymmetric around zero. The pattern of significance for the variables and their relative magnitudes was similar to that in specification 1. More importantly, the difference between the maximised log-likelihood for the probit and complementary log-log models was always less than 1%, suggesting little difference in the suitability of the two models.

Additionally, specification 1 was re-run using explanatory variables recorded in period t rather than in period $t-1$. This introduces the issue of two-way causation, however, there is a big increase in sample size, from 3,268 to 5,650 observations. This increase in observations is because respondents only have to be present for one interview wave.

The average marginal effects for this contemporary sample are reported in Table 3.17. Compared to the average marginal effects for the paired-transition data in Table 3.17 there are some changes. The average marginal effects for the looking for paid work variable are now all positive, although none of them are statistically significant. Also, the magnitudes of the average marginal effects for the financial position variable drop, often to near zero.[43] However, importantly, the strong significance of the "OK to sometimes break the law" variable is repeated.

Under-reporting and attrition

As already mentioned, a concern with any econometric model of crime is under-reporting. The OCJS allowed respondents to answer "Don't know" and "Don't want to answer" to each offending question. A control for under-reporting would recognise that offenders might strategically answer "Don't know" or "Don't want to answer" to avoid admissions of offending. It is difficult to think of a situation where genuine

[43]This may be because a successful offender can materially improve their financial position.

non-offenders would have an incentive not to report their non-offending behaviour. As a first step to controlling for this strategic answering, specification 4 re-runs specification 1 after re-coding responses of "Don't Know" and "Don't want to answer" as reports of offending. This re-coding led to 68 extra reports of Theft, 71 extra reports of Economic Crime and 27 extra reports of Economic Crime (ex. work and school theft).[44]

Table 3.16 shows that in this under-reporting specification, the magnitude and significance of the variables are generally reduced. Most noticeably, significance is reduced for smaller shifts in attitudes to breaking the law. Nevertheless, the shift from "Agree" to "Disagree" remains significant at the 5% level, and the shift to "Strongly disagree" remains significant at the 1% level.

Using a bivariate probit model would be another, more sophisticated, way to control for under-reporting. In the spirit of Heckman (1979), Greene (2008) describes how one of the two binary processes estimated in the bivariate probit model could be a control for sample selection. Here, the sample selection process would represent whether an individual answered affirmatively, i.e. "Yes" or "No", to the offending questions, or whether they answered "Don't Know" or "Don't want to answer". However, implementation of this model is left for further work. Also, the effectiveness of this approach may be limited due to only a small number of individuals not answering affirmatively.

[44]The slight rise in sample size for specification 4 occurs because in specifications 1, 2 and 3, non-offenders answering "Don't know" or "Don't want to answer" to an offending question were dropped from the sample.

An additional issue is attrition. Attrition may cause bias if respondents drop out of the sample due to factors other than those described by the independent variables. It is certainly possible that offenders, may drop out of the sample at a higher rate than non-offenders due to the former group's increased risk of jail. However, in the full 10-25 sample, the number of respondents confirmed as being in prison when a re-interview was attempted was very low, being 1, 4 and 1 respondents in 2004, 2005 and 2006 respectively. The full re-interview rates for the 10-25 sample were fairly high, being 74.5%, 83% and 85% in 2004, 2005 and 2006 respectively.[45]

To understand how offending reports varied with time spent in the sample, a sweep variable was included in all the estimations. For Theft and Economic Crime, interviewees reporting independent variables in their third sweep showed a statistically significant drop in p_{it} of 4 to 6 percentage points. This suggests that those more likely to offend did drop out for factors other than those measured by the independent variables. As further work, one could formally model the attrition process by using information available in sweep s to model the probability of respondents completing the survey in sweep $s+1$.

3.9.1. Controlling for zero-inflation

The fixed effects logit model that uses the contemporary sample, and the bivariate probit model with partial observability that uses the paired-transitions sample, are

[45] These figures come from the survey documentation of Hamlyn et al (2005), Phelps et al (2006) and Phelps et al (2007). It should be noted that the figure for 2004 is lower because it excludes those cases, not used in our sub-sample, where some data was lost and a second interview was required. Including these cases would increase the 2004 re-interview rate to 81%.

now discussed. Further detail regarding these models' structures is provided in the Empirical Appendix. Both models attempt to overcome the issue of zero-inflation by controlling for the presence of those who never offend, i.e. the "honest" individuals. This interprets "integrity" as a broader characteristic than just the attitude to crime picked up by our integrity proxy. The aim is to understand with greater clarity the impact of time-varying characteristics, in particular economic circumstances, on the offending of those agents whose time invariant characteristics suggest they are at a high risk of offending.

As the fixed effects logit model is a conditional logit model, it requires there to be variation in the dependent variable, O_{it}. Hence, individuals included in its estimation must offend at least once within the sampling period.

This requirement for variation in the dependent variable significantly reduces the sample size. The sample size drops from 3,105 individuals in the main contemporary sample to only 236 for the Theft regression. As with standard fixed effects models, the estimation focuses on the within variation, i.e. the variation in the behaviour of each individual over time. However, after conditioning on variation in O_{it}, over 40% of respondents were in the sample for only two waves. This limits the variation in the independent variables.

The consistent lack of significance for the independent variables shown in Table 3.18 is, therefore, perhaps unsurprising.[46] The only consistently significant variable

[46]For comparison, the co-efficients from logit regressions using the contemporary sample are also reported in Table 3.18.

is the dummy for taking Class A drugs. However, the lack of statistical significance could also support a sorting story. Once an individual's time-invariant characteristics (both observable and unobservable) have been controlled for, other factors no longer have strong relationships with offending. Individuals sort according to their fixed characteristics to be either a criminal or a non-criminal.

The bivariate probit model with partial observability was introduced by Poirier (1980). It models the observed binary outcome - to offend or not offend - as the outcome of two correlated but unobserved binary processes. In the current setting, the first unobserved binary process is whether a respondent is an "honest" type or not. The second binary process is interpreted as whether or not a respondent's economic circumstances would induce someone "dishonest" to offend. Only if an individual is both "dishonest" and their circumstances make it attractive to offend, will offending be observed.

From Poirier's original work, it is known that identification can be problematic. Identification appears to be an issue in the current setting. Estimation was only possible for Theft and Economic Crime, and only if no independent variables were common to both the "honest/dishonest" regression and the economic circumstances regression. As such, all the time-invariant variables were used to estimate the "honest/dishonest" regression and all the time-varying variables were used to estimate economic circumstances regression. Hence, one obtains the average marginal effects of the time-varying variables conditional on the time-invariant variables indicating that a respondent is "dishonest".

The estimation results for this model are shown in Table 3.19. Considering the conditional average marginal effects, there is a general lack of significance for the time varying characteristics. This again fits a sorting story where a changing environment has only a weak relationship with offending. It is further evidence that within the OCJS sample the proportion of "unfortunates" compared to the "criminally inclined" appears low. The only variable with conditional average marginal effects statistically significant for both Theft and Economic Crime was having friends in trouble with the police. Also, for Economic Crime, conditional on being "dishonest", taking drugs was associated with an increase in the probability of offending. However, given the estimation issues encountered and the very specific model specification used, these results should be treated with a degree of caution.

3.9.2. Further Work

There are a range of possibilities for further work. The most interesting is to investigate further the relationship between asset holdings, a binding liquidity constraint and economic crime. In the model individuals only commit crime once their liquidity constraint binds, i.e. $A = 0$. When unemployed, low-integrity individuals will run down their asset holdings before offending. As such, a logical hypothesis is that as unemployment duration increases, individuals become more likely to offend. The information in the OCJS data on unemployment duration is too limited for this type of analysis.

Two alternative datasets present themselves for this future work. One is the JUVOS cohort, which is a 5% sample of those claiming unemployment benefit in the

UK. This dataset includes the destination of those leaving the claimant count between 1996 and 2006. The possible destinations include going to prison or appearing in court.[47] The other potential dataset is the US's National Longitudinal Survey of Youth 97 (NLSY97). The NLSY97 is a general panel survey. It includes more labour market information than the OCJS, but lacks questions regarding attitude to crime.

The longer sampling periods of these studies also allow the theoretical model to be considered in an environment including business cycle fluctuations. Most significantly, this would help to identify whether the apparent low number of "unfortunates" observed in the OCJS is due to the economic environment when the sampling took place, or a more general empirical result.

Staying with the OCJS data, it seems sensible to run multinomial probit models to obtain further information regarding the determinants of attitude to crime, employment status and financial position. The purpose is twofold. Firstly, it may suggest instruments that could be used to address any concerns about endogeneity in the estimations. Secondly, by identifying variables linked with attitude to crime, it should provide information about alternative integrity proxies which could be used in other, less detailed, datasets. As such, estimating a multinomial probit model for attitude to crime would be a useful precursor to any work using the NLSY97.

Lastly, a number of further robustness checks could be carried out. In particular, information regarding the frequency of offending and the monetary value of items

[47]This dataset has been suggested by Prof. Eric Smith.

stolen could prove important. The former would identify the number of prolific offenders within the sample. The latter would identify the seriousness of the crimes committed.

Another extension could be to use the geographic information identifying a respondent's PFA to link the survey data with other contextual information about areas. Incorporating information regarding labour market conditions could prove valuable. Such data could help identify individuals' expectations regarding the job finding rate and the wages available. If expectations of the returns to job search are low, the theoretical model suggests offending will appear relatively attractive.

3.10. Conclusion

Both the theoretical and empirical sections of the paper highlight the interplay between personal characteristics and economic circumstances that determine individuals' criminal decisions. In a dynamic framework, the optimal crime, job search, gambling and saving decisions of heterogeneous agents are derived. It is shown that an individual's aversion to crime is key to their criminal decision, and to whether employment status has an impact on this criminal decision.

In broad terms, the data provides support for this view. The results show that fixed personal characteristics and the immediate social environment are more important than employment status and financial position in determining offending behaviour. This fits with a notion of individuals sorting by integrity. High-integrity "honest" agents choose never to offend and low-integrity agents, the "criminally inclined", offend regardless of employment status.

In the OCJS data, the prevalence of workplace theft and the lack of relationship between unemployment and offending suggest that the "criminally inclined" are dominant amongst the offenders observed. In contrast, those with slightly higher integrity, the "unfortunates", whose offending behaviour depends on employment status, seem rare. Either this group is inherently small, or, the benign labour market conditions in 2003-2006 meant that these individuals were employed, had assets remaining or perceived good future earnings opportunities. It is the unusual detail of the OCJS data that makes these conclusions possible.

In conclusion, this chapter provides a rich theoretical model in which the heterogeneity of individuals and labour market conditions combine to determine individuals' choice between legitimate employment and crime. Many of the insights are novel, such as the relationship between asset holdings and crime, or provide alternative explanations for existing empirical relationships, such as the value of gambling to otherwise risk-averse offenders. The empirical analysis uses the richness of the OCJS to explore the theoretical framework highlighting, in particular, the link between individuals' initial attitude towards criminal activity and subsequent offending. Taken together, the model and data emphasise that any relationship between employment status and offending is likely to be complex. Not only do they emphasise that only a sub-section of the general population has the necessary inclination to offend, but also that some individuals will offend both when unemployed and employed. Lastly, the chapter provides avenues for further research, most notably, investigating the prediction of a positive relationship between unemployment duration and offending.

3.11. Technical Appendix

Proof of Theorem 3.1. The text has characterised optimal behaviour for $A \in [0, A^P]$. For the interval $A \in (A^P, A^R)$ the agent chooses $s = 0$ and $z = 0$, whilst optimal consumption smoothing implies $c = c^U(A^P)$ in this region. Thus, there is perfect consumption smoothing but $\dot{A} < 0$ implies the agent switches to job search, $s = 1$, and the consumption rule, $c^U(.)$, once $A \leq A^P$. As c does not change, $V^U(.)$ has a constant slope $u'(c^U)$ in this region.

A^R is identified where $b + rA^R = c^U(A^P)$. At $A = A^R$, the agent consumes $c = c^U(A^P)$ indefinitely; i.e. $\dot{A} = 0$ and the worker is sufficiently rich that never looking for work is an absorbing state. For $A > A^R$, the agent is retired: they choose $s = 0$ and $c^U = b + rA$. As $V^U = \frac{u(b+rA)}{r}$, V^U is increasing and concave.

As V^U is increasing and concave, the NCC is satisfied for all $A > 0$ whilst unemployed. Further, as $V^E(A) = \frac{u(w+rA)}{r} > V^U(A)$ and $\frac{dV^E}{dA} < \frac{dV^U}{dA}$ for all $A \geq 0$, it follows that the NCC_E holds for all $A \geq 0$. Thus consuming $c^E = w + rA$ while employed is indeed optimal. ■

Proof of Theorem 3.2. We first establish a solution for A^S exists and is unique. The LHS of (3.16) is a concave function of A^S whose maximum occurs at $\frac{b+z^U-w}{r}$. Furthermore, at this maximum, the LHS of (3.16) is:

$$\frac{u(b+z^U)}{r} - \frac{(b+z^U-w)}{r}u'(b+z^U)$$

As Proposition 3.1 and $z^E > 0$ imply $b + z^U - w > 0$, this latter term is a decreasing function of z^U. Hence:

$$\frac{u(b+z^U)}{r} - \frac{(b+z^U-w)}{r}u'(b+z^U) >$$

$$\frac{u(w+z^E)}{r} - \frac{z^E}{r}u'(w+z^E) = V^E(0)$$

by (3.10) and (3.11). Thus, strict concavity of $u(.)$ and continuity imply there exist two solutions for A^S satisfying (3.16). The smaller solution implies $A^S < \frac{b+z^U-w}{r}$, and, thus, $w + rA^s < b + z^U$ which is not the relevant case (consumption would then decrease for some A and as V^E is not then concave, the solution is not consistent with fair lotteries). Instead, a unique solution for A^S exists which satisfies (3.16) and $A^S > \frac{b+z^U-w}{r}$.

Optimal consumption smoothing implies that when employed, an agent with $A < \frac{z^E}{r}$ consumes $c^E = w_i + z^E$ and A will fall over time until $A = 0$. At $A = 0$, the agent switches to crime. For $A \geq \frac{z^E}{r}$, the agent instead consumes $w + rA$ in perpetuity and so never commits crime. As the solution for A^S implies $A^S > \frac{b+z^U-w}{r}$, and as $b + z^U > w + z^E$ from Proposition 3.1, we therefore have $A^S > \frac{z^E}{r}$: an employed agent with A^S never commits crime.

Finally, note the parameter space for the "criminally inclined" implies $V^E(0) - V^U(0) \geq \frac{d}{\lambda}$. It follows from (3.14) that:

$$V^E(A^S) - V^U(A^S) = V^E(0) - V^U(0)$$

and thus $V^E(A) - V^U(A) \geq \frac{d}{\lambda}$ for all $A \in [0, A^S]$. Thus $s = 1$ is optimal at $A = A^S$. The arguments used to characterise optimal behaviour for $A > 0$ in the proof of Theorem 3.1, now characterise optimal behaviour here, when $A > A^S$ and the initial value $c^U(A^S) = b + z^U$ at $A = A^S$. ■

3.12. Empirical Appendix

Number of Individuals	Percentage of Individuals	Paired-Transitions
551	27.50	. . 1
450	22.46	. 1 1
305	15.22	1 1 1
257	12.82	. 1 .
237	11.83	1 . .
184	9.18	1 1 .
20	1.00	1 . 1
2,004	**100.01**	**-**

Note: The percentage does not sum to 100% due to rounding error.

Year	Number of Paired-Transitions	Percentage of Paired-Transitions
2003-2004	746	22.83
2004-2005	1,196	36.60
2005-2006	1,326	40.58
Total:	**3,268**	**100.01**

Note: The percentage does not sum to 100% due to rounding error.

Table 3.9: Structure of the unbalanced panel and number of paired-transitions by year.

"I like taking risks in life"	Non-Offenders (Economic Crime)	Offenders (Economic Crime)	Paired-Transition Sample	Contemporary Sample
% Agree strongly	5.49	12.29	6.61	6.64
% Agree slightly	48.19	58.66	49.91	49.31
% Disagree slightly	29.70	21.60	28.37	28.67
% Disagree strongly	14.87	6.52	13.49	13.79
% Don't know/Refused	1.76	0.93	1.62	1.59
% Total[1]	**100.01**	**100.00**	**100.00**	**100.00**

Note: "Paired-Transition Sample" refers to a sample where respondents answered all questions regarding independent variables in period t-1 and all questions regarding dependent variables in period t. "Contemporary Sample" refers to a sample where respondents answered all questions for both independent and dependent variables in period t. All percentages have N as their base and, other than for "Contemporary Sample", refer to period t-1. The breakdown by offending refers to the "Paired-Transition Sample" with the Offender/Non-Offender classification determined by responses to offending questions in period t.

[1] Values that do not sum to 100% are due to rounding error.

Table 3.10: Responses to "I like taking risks in life".

Financial Assessment	Non-Offenders (Economic Crime)	Offenders (Economic Crime)	Paired-Transition Sample	Contemporary Sample
% Managing quite well	67.15	65.74	66.92	66.71
% Just getting by	28.38	27.37	28.21	28.48
% Getting into difficulties	3.04	5.21	3.40	3.70
% Don't know/Refused	1.43	1.68	1.47	1.12
% Total[1]	**100.00**	**100.00**	**100.00**	**100.01**

Note: "Paired-Transition Sample" refers to a sample where respondents answered all questions regarding independent variables in period t-1 and all questions regarding dependent variables in period t. "Contemporary Sample" refers to a sample where respondents answered all questions for both independent and dependent variables in period t. All percentages have N as their base and, other than for "Contemporary Sample", refer to period t-1. The breakdown by offending refers to the "Paired-Transition Sample" with the Offender/Non-Offender classification determined by responses to offending questions in period t.

[1] Values that do not sum to 100% are due to rounding error.

Table 3.11: Respondents' assessments of their financial position.

Activity in Past Week	Respondent				Household Reference Person[1]			
	Non-Offenders (Economic Crime)	Offenders (Economic Crime)	Paired-Transition Sample	Contemporary Sample	Non-Offenders (Economic Crime)	Offenders (Economic Crime)	Paired-Transition Sample	Contemporary Sample
% Going to school (including on holiday)	4.10	3.54	4.01	1.22	0.18	0.19	0.18	0.09
% Going to college (includes 6th form and university) full-time (including on holiday)	36.80	39.66	37.27	29.82	3.70	2.61	3.52	5.17
% In paid work - higher managerial, administrative and professional occupations	10.18	7.64	9.76	13.79	36.21	37.06	36.35	35.03
% In paid work - intermediate occupations	11.17	7.82	10.62	13.04	16.11	16.20	16.13	16.00
% In paid work - routine and manual occupations	23.76	30.17	24.82	27.27	27.68	29.80	28.03	27.86
% On a government scheme for employment training	0.73	0.93	0.76	0.83	0.18	0.00	0.15	0.19
% Doing unpaid work for a family business	0.04	0.00	0.03	0.04	0.07	0.00	0.06	0.09
% Waiting to take up paid work already obtained	0.55	0.74	0.58	0.69	0.18	0.74	0.28	0.32
% Looking for paid work or a government training scheme	3.63	2.79	3.49	3.86	1.43	0.56	1.29	1.47
% Intending to look for work but prevented by temporary sickness or injury	0.48	1.12	0.58	0.71	0.44	0.74	0.49	0.37
% Permanently unable to work because of long-term sickness or disability	0.26	0.37	0.28	0.34	2.38	1.86	2.29	2.16
% Retired from paid work	0.00	0.19	0.03	0.02	3.59	4.10	3.67	3.93
% Looking after home or family	6.85	2.79	6.18	6.90	7.36	5.21	7.01	6.65
% Doing something else	1.46	2.23	1.59	1.47	0.48	0.93	0.55	0.67
Total[2]	**100.01**	**99.99**	**100.00**	**100.00**	**99.99**	**100.00**	**100.00**	**100.00**

Note: "Paired-Transition Sample" refers to a sample where respondents answered all questions regarding independent variables in period t-1 and all questions regarding dependent variables in period t. "Contemporary Sample" refers to a sample where respondents answered all questions for both independent and dependent variables in period t. All percentages have N as their base and, other than for "Contemporary Sample", refer to period t-1. The breakdown by offending refers to the "Paired-Transition Sample" with the Offender/Non-Offender classification determined by responses to offending questions in period t.

[1] Household Reference Person is defined as the person who is the owner/renter of the household's accomodation. If the accomodation is in joint names it is the person with the highest income. If incomes are the same it is the older person.

[2] Values that do not sum to 100% are due to rounding error.

Table 3.12: Employment status of respondent and HRP.

Independent Variables	Categories used in Regression	Notes	"Null" Category
Managing on income	Just getting by, Getting into difficulties and Don't Know/Refused		Managing quite well
Employment status	In full time education/training, Intermediate occupations, Routine and manual occupations, Looking for work/training, Looking after home/family, Other (specified) and Doing something else	Formed from responses to several questions. The first question asks individuals to specify, from a list of 12 options, their main activity in the past week. Options with few responses, such as unable to work due to sickness, were grouped into "Other (specified)". For those individuals replying that they were in paid work a variety of other questions were asked regarding their job title, responsibilities and employer. Respondents' occupational level was then classified according to the National Statistics Socio-economic Classification. This classification was provided as part of the original dataset.	Higher managerial, administrative and professional occupations
Household income group	Under £4,999, £5,000-£14,999, £15,000-£24,999, £25,000-£34,999, £35,000-£44,999, Don't know/Refused	Original question involved 12 income groups. Respondents were provided with annual, monthly and weekly income equivalents.	£45,000+
Housing tenure	Paying off mortgage and Renting		Own outright/Live rent-free
Highest educational qualification obtained	A/AS-Levels or equivalent, GCSE grades A*-C/Trade apprenticeships or equivalent, GCSE grades D-G or equivalent	Compared to other variables a relatively high number of respondents had this data marked as missing.	A higher education qualification
Gender	Male		Female
Relationship status	Single	Respondents who were separated but not divorced are grouped as being "In a relationship".	In relationship
Drug use in past year	Taken 'Class A' drugs, Taken drugs (not 'Class A')	Class A' drugs include cocaine. Cannabis is not a 'Class A' drug. It has a lower (less serious) classification.	Not taken illegal drugs
Alcohol consumption	Most days, Once or twice a week, 1-3 times a month		Less often than "Once a month"
Has biological children	Yes		No
Lives with parents	Yes		No
Friends in trouble with the police in past year	Yes	Exact question wording: "Thinking about your closest friends. About how many of them, if any, have been in trouble with the police in the last 12 months?" Respondents could answer: none, a few, quite a lot, nearly all, all. All responses other than "None" are classified as "Yes".	No

Independent Variables	Categories used in Regression	Notes	"Null" Category
Age	Treated as continuous variable		-
Sometimes OK to break the law	Strongly agree, Neither agree nor disagree, Disagree, Strongly disagree	During interview asked prior to questions regarding crime victimisation and offending.	Agree
OK to steal if you are very poor	Strongly agree, Neither agree nor disagree, Disagree, Strongly disagree	During interview asked prior to questions regarding crime victimisation and offending.	Agree
OK to steal from somebody rich	Strongly agree, Neither agree nor disagree, Disagree, Strongly disagree	During interview asked prior to questions regarding crime victimisation and offending.	Agree
OK to steal from shop	Strongly agree, Neither agree nor disagree, Disagree, Strongly disagree	During interview asked prior to questions regarding crime victimisation and offending.	Agree
Household size	1 person, 2 people and 6+ people		3-5 people
Police Force Area	Dummies for the 41 Police Force Areas (other than Metropolitan/City of London) in England and Wales	Police Force Areas often, but not always, cover the same areas as county councils. Some police forces cover multiple counties and some cover specific conurbations.	Metropolitan/City of London
Belongs to a religious group	Yes	The survey does not ask whether individuals are active in their faith e.g. actually go to church.	No
Member of ethnic minority	Yes	Ethnic majority defined as white from any background.	No
Interview wave	Wave 2, Wave 3 and Wave 4	Wave 4 only features in the "Contemporary" regressions. In the baseline regressions as the lag of the wave is used Wave 3 is the last wave of data used for the independent variables.	Wave 1
Victim of crime in past year	Yes	Victimisation questions cover the same range of offences as the offending questions. Crimes against the respondent and the physical household are considered.	No
Safety of walking alone in area after dark	Fairly safe, Fairly unsafe and Very unsafe	Exact question wording: "How safe would you feel walking alone in this area after dark. Would you feel..."	Very safe

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Independent Variables	Categories used in Regression	Notes	"Null" Category
Belongs to sports club/gym	Yes	Original survey question identified 9 different types of groups/clubs/organisations to which a respondent could belong. Respondents could also specify other types of groups to which they were members.	No
Answered all crime questions truthfully	Yes	Exact question wording: "When you answered the questions about committing crimes, how truthful were you?" Only respondents agreeing with the statement "I answered ALL questions truthfully" are treated as "Yes".	No
Interview sweep	Sweep 2, Sweep 3 and Sweep 4	Sweep 4 only features in the "Contemporary" regressions. In the baseline regressions, as the lag of the sweep is used, Sweep 3 is the last sweep of data used for the independent variables.	Sweep 1
Parents spent time in prison (before 1st interview)	Yes, Don't Know/Refused		No
Ever expelled (before 1st interview)	Yes		No
Ever arrested (before 1st interview)	Yes	Note that in the UK being arrested does not necessarily lead to a court appearance or even being charged with an offence.	No
Ever sentenced (before 1st interview)	Yes		No
Ever spent time in prison (before 1st interview)	Yes		No
Ever sought help for mental health problems (before 1st interview over age 16)	Yes	Question only asked of those aged over 16.	No
Ever committed offence (before 1st interview)	Yes	The classification of offence varies by regression to match the dependent variable being estimated. For the Economic Crime variables the relevant "Ever committed offence" variables do not include the offences of credit card fraud and selling stolen goods. This is because only those over the age of 18 were asked about these offences.	No

Table 3.13: Description of the independent variables used.

Predicted Probability of Offending Thresholds	Percentage of Predictions Below Threshold					
	Specification 1			Specification 2		
	Theft	Economic Crime	Economic Crime (ex. work and school theft)	Theft	Economic Crime	Economic Crime (ex. work and school theft)
Below 0.01	12.09	3.89	12.33	18.18	5.72	14.44
Below 0.05	43.15	27.48	45.62	51.01	32.71	48.13
Below 0.10	63.92	48.07	64.14	66.62	51.41	65.15
Below 0.25	88.4	77.63	86.51	86.44	76.81	85.95
Below 0.50	98.53	94.31	96.94	97.86	93.18	96.14
Below 0.75	100.00	99.24	99.51	100.00	99.02	99.57
Below 0.90	100.00	99.91	99.97	100.00	99.88	99.97
Below 1.00	100.00	100.00	100.00	100.00	100.00	100.00

Note: The cumulative percentages are as a percentage of the total number of predictions. The total number of predictions is 3,268.

Table 3.14: Distributions of predicted offending probabilities.

Independent Variable	Successful Graduate	Single mother	A-Level Student	Family Man	Trouble maker	Average Joe
Managing on income	Managing quite well	Getting into difficulties	Managing quite well	Just getting by	Just getting by	Managing quite well
Employment status	Higher managerial, administrative and professional occupations	Looking after home/family	In full time education/ training	Intermediate occupation	Looking for paid work/ training scheme	Routine/ manual occupation
Sometimes OK to break the law	Disagree	Neither agree nor disagree	Neither agree nor disagree	Strongly disagree	Agree	Neither agree nor disagree
Household income group	£35,000-£44,999	Under £5,000	£25,000-34,999	£25,000-34,999	£15,000-24,999	£15,000-24,999
Housing tenure[1]	Rent	Rent	Mortgage	Rent	Mortgage	Mortgage
Highest educational qualification obtained	Higher education qualification	GCSEs grades D-G or equivalent	GCSEs grades A*-C or equivalent	Higher education qualification	GCSEs grades A*-C or equivalent	A Levels or equivalent
Gender	Female	Female	Male	Male	Male	Male
Relationship status	In relationship	Single	Single	In relationship	Single	In Relationship
Drug use in past year	None	None	Yes - Not 'Class A'	None	Yes - 'Class A'	None
Alcohol consumption	Once or twice a week	Once to three times a month	Once or twice a week	Once or twice a week	Once or twice a week	Once or twice a week
Has biological children	No	Yes	No	Yes	No	No
Lives with parents	No	No	Yes	No	Yes	Yes
Friends in trouble with the police in past year	No	Yes	Yes	No	Refused to answer	No
Age	24	19	18	24	20	20
Household size	2 people	3-5 people	3-5 people	3-5 people	6+ people	3-5 people
Police Force Area	Metropolitan (London)	Greater Manchester	Nottinghamshire	Wiltshire	West Midlands	Northamptonshire
Belongs to a religious group	No	Yes	No	Yes	No	No
Member of ethnic minority	No	No	Yes	No	No	No
Interview wave	2	2	2	2	2	2
Victim of crime in past year	No	Yes	Yes	No	No	No
Safety of walking alone in area after dark	Fairly unsafe	Very unsafe	Fairly unsafe	Very unsafe	Fairly safe	Fairly safe
Belongs to sports club/gym	Yes	No	Yes	Yes	No	Yes
Answered crime questions truthfully	Yes	Yes	Yes	Yes	No	Yes
Interview sweep	2	2	2	2	2	2
Parents spent time in prison (before 1st interview)	No	Don't Know	No	No	Yes	No
Ever expelled (before 1st interview)	No	No	No	No	Yes	No
Ever arrested (before 1st interview)	No	No	No	No	Yes	No
Ever sentenced (before 1st interview)	No	No	No	No	No	No
Ever spent time in prison (before 1st interview)	No	No	No	No	No	No

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	Successful Graduate	Single mother	A-Level Student	Family man	Rogue	Average Joe
Ever sought help for mental health problems (before 1st interview over age 16)	No	Yes	No	No	No	No
Admits committing Economic Crime (before 1st interview)	No	No	No	No	Yes	No
Excluding prior offending variable						
Predicted probability of reporting Theft in following time period[2]	**0.042**	**0.032**	**0.229**	**0.019**	**0.263**	**0.173**
Predicted probability of reporting Economic Crime in following time period[2]	**0.054**	**0.107**	**0.195**	**0.039**	**0.607**	**0.257**
Predicted probability of reporting Economic Crime (ex. work and school theft) in following time period[2]	**0.014**	**0.093**	**0.170**	**0.010**	**0.502**	**0.106**
Including prior offending variable						
Predicted probability of reporting Theft in following time period[2]	**0.026**	**0.014**	**0.084**	**0.008**	**0.303**	**0.072**
Predicted probability of reporting Economic Crime in following time period[2]	**0.040**	**0.063**	**0.111**	**0.022**	**0.639**	**0.149**
Predicted probability of reporting Economic Crime (ex. work and school theft) in following time period[2]	**0.011**	**0.066**	**0.126**	**0.007**	**0.560**	**0.066**

[1] The statement "Mortgage" does not imply that the respondent has a mortgage. Instead it implies the household reference person pays a mortgage.

[2] These predicted probabilities should be treated with a degree of caution. The confidence intervals are very wide, often over twenty percentage points, indicating that the predictions are imprecise. Also recall that the model is not causal. The predicted probabilities are included for illustrative purposes only.

Table 3.15: Description of hypothetical individuals.

Independent Variable	Specification 3 - All Integrity Proxies: Theft	Specification 3 - All Integrity Proxies: Economic Crime	Specification 3 - All Integrity Proxies: Economic Crime (ex. work and school theft)	Specification 4 - Under-Reporting Control: Theft	Specification 4 - Under-Reporting Control: Economic Crime	Specification 4 - Under-Reporting Control: Economic Crime (ex. work and school theft)
Household just getting by on income	0.016	0.019	0.024*	0.008	0.009	0.021
	(0.013)	(0.015)	(0.013)	(0.013)	(0.015)	(0.013)
Household getting into difficulties on income	0.041	0.066*	0.044	0.045	0.074*	0.048
	(0.034)	(0.040)	(0.032)	(0.035)	(0.041)	(0.031)
Respondent employment status: intermediate occupation	-0.010	-0.014	-0.010	-0.008	-0.017	-0.017
	(0.023)	(0.029)	(0.026)	(0.024)	(0.029)	(0.026)
Respondent employment status: routine and manual occupations	0.028	0.026	0.013	0.026	0.020	0.008
	(0.021)	(0.026)	(0.023)	(0.021)	(0.026)	(0.023)
Respondent employment status: looking for paid work/training	-0.046*	-0.048	-0.028	-0.037	-0.043	-0.038
	(0.027)	(0.035)	(0.030)	(0.030)	(0.037)	(0.031)
Strongly agree: sometimes OK to break the law (1st interview)	-0.062*	-0.063	-0.057	-0.036	-0.048	-0.047
	(0.033)	(0.046)	(0.037)	(0.043)	(0.052)	(0.041)
Neither agree/disagree: sometimes OK to break the law (1st interview)	-0.031*	-0.035*	-0.037**	-0.028	-0.030	-0.037**
	(0.018)	(0.021)	(0.016)	(0.020)	(0.022)	(0.017)
Disagree: sometimes OK to break the law (1st interview)	-0.040**	-0.048**	-0.034**	-0.046**	-0.052**	-0.040**
	(0.018)	(0.020)	(0.016)	(0.018)	(0.020)	(0.016)
Strongly disagree: sometimes OK to break the law (1st interview)	-0.078***	-0.074***	-0.031	-0.098***	-0.092***	-0.046***
	(0.019)	(0.022)	(0.019)	(0.018)	(0.021)	(0.017)
Strongly agree: OK to steal if you are very poor (1st interview)[1]		-0.122	-0.073	-	-	-
		(0.147)	(0.117)			
Neither agree/disagree: OK to steal if you are very poor (1st interview)	-0.064*	-0.108***	-0.067**	-	-	-
	(0.035)	(0.040)	(0.032)			
Disagree: OK to steal if you are very poor (1st interview)	-0.045	-0.070*	-0.035	-	-	-
	(0.035)	(0.039)	(0.031)			
Strongly Disagree: OK to steal if you are very poor (1st interview)	-0.050	-0.074*	-0.056	-	-	-
	(0.037)	(0.042)	(0.035)			
Strongly agree: OK to steal from somebody rich (1st interview)	0.123	-0.123*	-0.115*	-	-	-
	(0.121)	(0.071)	(0.066)			
Neither agree/disagree: OK to steal from somebody rich (1st interview)	0.006	0.040	0.012	-	-	-
	(0.049)	(0.066)	(0.064)			
Disagree: OK to steal from somebody rich (1st interview)	0.008	0.008	-0.027	-	-	-
	(0.043)	(0.055)	(0.056)			
Strongly Disagree: OK to steal from somebody rich (1st interview)	0.006	0.004	-0.037	-	-	-
	(0.046)	(0.058)	(0.059)			
Strongly agree: OK to steal from shop (1st interview)	0.195	0.121	0.103	-	-	-
	(0.145)	(0.264)	(0.231)			
Neither agree/disagree: OK to steal from shop (1st interview)	0.068	-0.024	-0.059	-	-	-
	(0.069)	(0.083)	(0.069)			
Disagree: OK to steal from shop (1st interview)	-0.033	-0.078	-0.081	-	-	-
	(0.060)	(0.078)	(0.067)			
Strongly Disagree: OK to steal from shop (1st interview)	-0.053	-0.099	-0.083	-	-	-
	(0.063)	(0.081)	(0.070)			

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	Theft	Economic Crime	Economic Crime (ex. work and school theft)	Theft	Economic Crime	Economic Crime (ex. work and school theft)
Taken drugs in past year (not 'Class A')	0.088***	0.134***	0.106***	0.091***	0.138***	0.110***
	(0.015)	(0.018)	(0.015)	(0.015)	(0.018)	(0.016)
Taken 'Class A' drugs in past year	0.074***	0.160***	0.150***	0.088***	0.175***	0.155***
	(0.021)	(0.027)	(0.023)	(0.023)	(0.028)	(0.024)
Friends in trouble with police in past year	0.056***	0.076***	0.050***	0.062***	0.081***	0.057***
	(0.013)	(0.016)	(0.013)	(0.014)	(0.017)	(0.013)
Victim of crime in past year	0.028***	0.027**	0.022**	0.022*	0.020	0.024**
	(0.011)	(0.013)	(0.011)	(0.011)	(0.013)	(0.011)
Ever expelled (before 1st interview)	0.024	0.114**	0.106**	0.025	0.114**	0.097**
	(0.041)	(0.050)	(0.048)	(0.041)	(0.049)	(0.047)
Ever arrested (before 1st interview)	0.011	0.050**	0.051**	0.001	0.043*	0.049**
	(0.020)	(0.024)	(0.022)	(0.021)	(0.025)	(0.022)
Ever sent to prison (before 1st interview)	0.084	0.232	0.175	0.100	0.247	0.181
	(0.121)	(0.198)	(0.170)	(0.130)	(0.199)	(0.178)
Male	0.033***	0.063***	0.055***	0.030**	0.061***	0.056***
	(0.012)	(0.014)	(0.012)	(0.013)	(0.015)	(0.012)
Age	-0.002	-0.006	-0.007**	-0.004	-0.007*	-0.008**
	(0.003)	(0.004)	(0.003)	(0.003)	(0.004)	(0.003)
Ever sought help for mental health problems (before 1st interview over age 16)	0.043***	0.072***	0.056***	0.045***	0.074***	0.057***
	(0.016)	(0.019)	(0.015)	(0.016)	(0.019)	(0.016)
PFA: Derbyshire	-0.080**	-0.118***	-0.103***	-0.045	-0.085**	-0.103***
	(0.036)	(0.041)	(0.032)	(0.039)	(0.043)	(0.031)
PFA: Devon & Cornwall	-0.067*	-0.111***	-0.094***	-0.057	-0.099**	-0.080**
	(0.036)	(0.040)	(0.033)	(0.039)	(0.043)	(0.036)
PFA: Essex	-0.103***	-0.153***	-0.127***	-0.121***	-0.170***	-0.128***
	(0.035)	(0.039)	(0.027)	(0.035)	(0.040)	(0.027)
PFA: North Yorkshire	-0.109***	-0.155***	-0.100***	-0.075	-0.123**	-0.109***
	(0.033)	(0.041)	(0.036)	(0.049)	(0.055)	(0.035)
Sweep 3	-0.046**	-0.060***	-0.034*	-0.040**	-0.056**	-0.033*
	(0.018)	(0.022)	(0.019)	(0.020)	(0.024)	(0.019)
N	**3,253**	**3,268**	**3,268**	**3,341**	**3,341**	**3,341**
i	**1,995**	**2,004**	**2,004**	**2,025**	**2,025**	**2,025**
Log likelihood	**-875.94**	**-1,154.99**	**-888.54**	**-1,024.85**	**-1,287.21**	**-960.35**
Median predicted probability of offending report	**0.061**	**0.104**	**0.057**	**0.080**	**0.124**	**0.064**
p-value for joint test of managing on income H_0: =0	**0.370**	**0.297**	**0.028**	**0.267**	**0.308**	**0.111**
p-value for joint test of employment status H_0: =0	**0.068**	**0.182**	**0.313**	**0.037**	**0.073**	**0.286**
p-value for joint test of 'OK to break the law' H_0: =0	**0.001**	**0.021**	**0.133**	**0.000**	**0.000**	**0.069**

Notes: Cluster robust standard errors are given in parentheses. Significance levels: * 10% significance, ** 5% significance and *** 1% significance. The p-values reported test whether the marginal effects are jointly different from zero for the set of independent variables stated. Specification 4 uses the same independent variables as specification 1. However, in an attempt to control for under-reporting, in specification 4 responses of "Don't Know" and "Don't Want to Answer" to the offending questions have been recorded as reports of offending. This also explains the small increase in sample size as individuals reporting "Don't Know" and "Don't Want to Answer" to offending questions were dropped in the other specifications. Independent variables which were frequently significant at the 5% level or above but not shown here for brevity are: Household income: £35,000-£44,999 (positive); Drinks alcohol 1-3 times a month (positive); Household size:1 (negative); PFA: Dyfed Powys (negative); PFA: Hampshire (negative); PFA: West Mercia (negative); PFA: Wiltshire (negative); Walking alone in local area at night fairly unsafe (negative); Sports club/gym member (positive); and Not 100% truthful re: crime questions (positive). Many other independent variables were also significant in individual regressions at the 10% level or above.

[1] A co-efficient is not reported for "Strongly agree: OK to steal if you are very poor" since no respondent reporting this attitude reported an offence in the following period. As a result these observations were dropped from the regression. This also explains the drop in N and i for the "Theft" regression in specification 3.

Table 3.16: Average marginal effects for the baseline probits using specifications 3 and 4.

Independent Variable	Specification 1 - Baseline probit Theft	 Economic Crime	 Economic Crime (ex. work and school theft)
Household just getting by on income	0.011	0.004	-0.001
	(0.009)	(0.012)	(0.010)
Household getting into difficulties on income	0.000	-0.002	0.011
	(0.021)	(0.025)	(0.022)
Respondent employment status: intermediate occupation	0.009	0.008	0.004
	(0.014)	(0.019)	(0.017)
Respondent employment status: routine and manual occupations	0.025*	0.015	0.001
	(0.013)	(0.017)	(0.015)
Respondent employment status: looking for paid work/training	0.026	0.005	0.011
	(0.022)	(0.026)	(0.023)
Strongly agree: sometimes OK to break the law (1st interview)	-0.053*	-0.048	-0.003
	(0.030)	(0.047)	(0.044)
Neither agree/disagree: sometimes OK to break the law (1st interview)	-0.038***	-0.051***	-0.049***
	(0.014)	(0.017)	(0.014)
Disagree: sometimes OK to break the law (1st interview)	-0.061***	-0.073***	-0.046***
	(0.013)	(0.016)	(0.013)
Strongly disagree: sometimes OK to break the law (1st interview)	-0.085***	-0.094***	-0.056***
	(0.014)	(0.017)	(0.014)
Taken drugs in past year (not 'Class A')	0.079***	0.118***	0.090***
	(0.012)	(0.014)	(0.012)
Taken 'Class A' drugs in past year	0.131***	0.227***	0.190***
	(0.017)	(0.021)	(0.019)
Friends in trouble with police in past year	0.042***	0.066***	0.042***
	(0.011)	(0.013)	(0.010)
Victim of crime in past year	0.046***	0.054***	0.039***
	(0.008)	(0.009)	(0.008)
Parents spent time in prison (before 1st interview)	0.065*	0.099**	0.083**
	(0.039)	(0.042)	(0.037)
Ever expelled (before 1st interview)	0.002	0.063*	0.058*
	(0.026)	(0.033)	(0.030)
Ever arrested (before 1st interview)	-0.007	0.016	0.024*
	(0.013)	(0.016)	(0.014)
Ever sent to prison (before 1st interview)	0.064	0.119	0.029
	(0.067)	(0.076)	(0.056)
Male	0.030***	0.053***	0.042***
	(0.009)	(0.012)	(0.010)
Age	-0.004*	-0.008***	-0.008***
	(0.002)	(0.003)	(0.002)

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	Theft	Economic Crime	Economic Crime (ex. work and school theft)
Ever sought help for mental health problems (before 1st interview over age 16)	0.026**	0.042***	0.037***
	(0.011)	(0.013)	(0.011)
PFA: Derbyshire	-0.002	-0.023	-0.047
	(0.037)	(0.040)	(0.030)
PFA: Devon & Cornwall	-0.064***	-0.068**	-0.049*
	(0.024)	(0.031)	(0.027)
PFA: Essex	-0.075***	-0.134***	-0.108***
	(0.027)	(0.029)	(0.020)
PFA: North Yorkshire	-0.059*	-0.101**	-0.062*
	(0.034)	(0.041)	(0.035)
Sweep 3	0.008	-0.002	-0.013
	(0.013)	(0.016)	(0.014)
N	**5,650**	**5,650**	**5,650**
i	**3,105**	**3,105**	**3,105**
Log likelihood	**-1,463.42**	**-2,069.76**	**-1,609.46**
Median predicted probability of offending report	**0.055**	**0.108**	**0.064**
p-value for joint test of managing on income H_0: =0	**0.652**	**0.981**	**0.949**
p-value for joint test of employment status H_0: =0	**0.356**	**0.902**	**1.000**
p-value for joint test of 'OK to break the law' H_0: =0	**0.000**	**0.000**	**0.001**

Notes: Cluster robust standard errors are given in parentheses. Significance levels: * 10% significance, ** 5% significance and *** 1% significance. The p-values reported test whether the marginal effects are jointly different from zero for the set of independent variables stated. Independent variables which were frequently significant at the 5% level or above but not shown here for brevity are: PFA: Gwent (negative); PFA: South Wales (negative); Wave 2 (positive); and Sports club/gym member (positive). Many other independent variables were also significant in individual regressions at the 10% level or above.

Table 3.17: Average marginal effects for the baseline probits (specification 1) using the contemporary sample.

Fixed Effects Logit (Conditional Logit) Model

The fixed effects logit model removes all the characteristics of individuals that are fixed through time, including those which are unobservable. Using the fixed effects logit model, a consistent estimator of β can be obtained without any assumptions regarding the relationship between individuals' fixed characteristics and the other explanatory variables. As Wooldridge (2002) describes, this is possible due to the logit link function's specific functional form. To understand why this is possible, firstly, denote the individual fixed effect α_i and $\mathbf{x}_i = (\mathbf{x}_{i1}, ..., \mathbf{x}_{iT})$. The next step is to find the joint distribution of $\mathbf{O}_i \equiv (O_{i1}, ..., O_{iT})'$ conditional on $\mathbf{x}_i$, α_i and $\eta_i = \sum_{t=1}^{T} O_{it}$ (η_i is the total number of offences reported within the sampling period).

The following is adapted from Wooldridge (2002) with changed notation. It demonstrates the key insight that the conditional distribution described does not depend on α_i and that, hence, $\boldsymbol{\beta}$ can be estimated using conditional maximum likelihood techniques.

Consider the simplest case of $T = 2$. When $\eta_i = 0$ or $\eta_i = 2$, the conditional distribution of $(O_{i1}, O_{i2})'$ given η_i cannot be informative for estimating $\boldsymbol{\beta}$ because the value of η_i completely determines the value of $\mathbf{O}_i$. Hence, to estimate $\boldsymbol{\beta}$, only cases where there is variation in O_{it} are used, i.e. $\eta_i = 1$. This means, by definition, only those individuals who offend at some point during the sampling period will be included in the estimation and the most persistent offenders will be excluded.

Suppose the probability of offending in period 2 is being estimated. Assuming conditional independence, so that O_{i2} is independent of O_{i1}, and after conditioning on $\mathbf{x}_i$ and α_i it is possible to write:

$$P\left(O_{i2} = 1 | \mathbf{x}_i, \alpha_i, \eta_i = 1\right) = \frac{P\left(O_{i2} = 1 \cap \eta_i = 1 | \mathbf{x}_i, \alpha_i\right)}{P\left(\eta_i = 1 | \mathbf{x}_i, \alpha_i\right)}$$

$$= \frac{P\left(O_{i2} = 1 | \mathbf{x}_i, \alpha_i\right) P\left(O_{i1} = 0 | \mathbf{x}_i, \alpha_i\right)}{P\left(O_{i2} = 1 \cap O_{i1} = 0 | \mathbf{x}_i, \alpha_i\right) + P\left(O_{i2} = 0 \cap O_{i1} = 1 | \mathbf{x}_i, \alpha_i\right)}$$

As the logit function is being used:

$$P\left(O_{it}|\mathbf{x}_i, \alpha_i\right) = \frac{\exp\left(\mathbf{x}_{it}'\boldsymbol{\beta} + \alpha_i\right)}{1 + \exp\left(\mathbf{x}_{it}'\boldsymbol{\beta} + \alpha_i\right)}$$

which, in turn, means:

$$\frac{P\left(O_{i2} = 1|\mathbf{x}_i, \alpha_i\right) P\left(O_{i1} = 0|\mathbf{x}_i, \alpha_i\right)}{P\left(O_{i2} = 1 \cap O_{i1} = 0|\mathbf{x}_i, \alpha_i\right) + P\left(O_{i2} = 0 \cap O_{i1} = 1|\mathbf{x}_i, \alpha_i\right)} =$$

$$\left(\frac{\exp\left(\mathbf{x}_{i2}'\boldsymbol{\beta} + \alpha_i\right)}{1 + \exp\left(\mathbf{x}_{i2}'\boldsymbol{\beta} + \alpha_i\right)} \times \frac{1}{1 + \exp\left(\mathbf{x}_{i1}'\boldsymbol{\beta} + \alpha_i\right)}\right) \times$$

$$\left[\begin{array}{c} \left(\frac{\exp\left(\mathbf{x}_{i2}'\boldsymbol{\beta}+\alpha_i\right)}{1+\exp\left(\mathbf{x}_{i2}'\boldsymbol{\beta}+\alpha_i\right)} \times \frac{1}{1+\exp\left(\mathbf{x}_{i1}'\boldsymbol{\beta}+\alpha_i\right)}\right) + \\ \left(\frac{1}{1+\exp\left(\mathbf{x}_{i2}'\boldsymbol{\beta}+\alpha_i\right)} \times \frac{\exp\left(\mathbf{x}_{i1}'\boldsymbol{\beta}+\alpha_i\right)}{1+\exp\left(\mathbf{x}_{i1}'\boldsymbol{\beta}+\alpha_i\right)}\right) \end{array}\right]^{-1}$$

Cancelling all the denominators gives:

$$P\left(O_{i2} = 1|x_i, \alpha_i, \eta_i = 1\right) = \frac{\exp\left(\mathbf{x}_{i2}'\boldsymbol{\beta} + \alpha_i\right)}{\exp\left(\mathbf{x}_{i2}'\boldsymbol{\beta} + \alpha_i\right) + \exp\left(\mathbf{x}_{i1}'\boldsymbol{\beta} + \alpha_i\right)}$$

$$= \frac{\exp\left(\mathbf{x}_{i2}'\boldsymbol{\beta}\right)}{\exp\left(\mathbf{x}_{i2}'\boldsymbol{\beta}\right) + \exp\left(\mathbf{x}_{i1}'\boldsymbol{\beta}\right)} = \frac{\exp\left[\left(\mathbf{x}_{i2}' - \mathbf{x}_{i1}'\right)\boldsymbol{\beta}\right]}{1 + \left[\exp\left(\left(\mathbf{x}_{i2}' - \mathbf{x}_{i1}'\right)\boldsymbol{\beta}\right)\right]}$$

and

$$P\left(O_{i1} = 1|\mathbf{x}_i, \alpha_i, \eta_i = 1\right) = \frac{1}{1 + \left[\exp\left(\left(\mathbf{x}_{i2}' - \mathbf{x}_{i1}'\right)\boldsymbol{\beta}\right)\right]}$$

The probability of offending in each period depends only on the first differences of the independent variables. For higher T, equivalent manipulations can be performed. Since the resulting expressions do not contain α_i, the individual fixed effects are not estimated. Also, as the first differences are being used, coefficients for the time-invariant independent variables are not identified.[48]

[48] Additionally, the Sweep variable has to be dropped. This is because, by definition, one period changes in the Wave variable and the Sweep variable are identical.

That α_i drops out of the estimation means only probabilities of offending conditional on η_i can be estimated and marginal effects cannot be computed. Due to this, Table 3.18 reports coefficients rather than marginal effects.

	Contemporary Logit			Contemporary Fixed Effects Logit		
Independent Variable	**Theft**	**Economic Crime**	**Economic Crime (ex. work and school theft)**	**Theft**	**Economic Crime**	**Economic Crime (ex. work and school theft)**
Household just getting by on income	0.147	0.024	-0.040	-0.252	-0.192	-0.187
	(0.126)	(0.105)	(0.123)	(0.328)	(0.250)	(0.317)
Household getting into difficulties on income	-0.072	-0.030	0.093	-1.403	-0.791	-0.139
	(0.288)	(0.234)	(0.262)	(1.248)	(0.697)	(0.769)
Respondent employment status: intermediate	0.110	0.069	0.072	0.354	0.206	0.300
	(0.217)	(0.173)	(0.215)	(0.597)	(0.435)	(0.621)
Respondent employment status: routine and manual occupations	0.303	0.122	0.034	0.119	-0.183	-0.100
	(0.187)	(0.152)	(0.191)	(0.542)	(0.377)	(0.513)
Respondent employment status: looking for paid work/training	0.296	0.047	0.172	2.139	0.766	0.692
	(0.282)	(0.232)	(0.274)	(1.729)	(0.621)	(0.871)
Strongly agree: sometimes OK to break the law (1st interview)	-0.566	-0.359	-0.035	-	-	-
	(0.389)	(0.371)	(0.402)			
Neither agree/disagree: sometimes OK to break the law (1st interview)	-0.381**	-0.354***	-0.488***	-	-	-
	(0.154)	(0.125)	(0.145)			
Disagree: sometimes OK to break the law (1st	-0.710***	-0.580***	-0.492***	-	-	-
	(0.151)	(0.122)	(0.139)			
Strongly disagree: sometimes OK to break the law (1st interview)	-1.158***	-0.796***	-0.619***	-	-	-
	(0.202)	(0.146)	(0.164)			
Taken drugs in past year (not 'Class A')	1.017***	0.964***	1.052***	0.359	0.280	0.394
	(0.130)	(0.103)	(0.121)	(0.408)	(0.282)	(0.365)
Taken 'Class A' drugs in past year	1.444***	1.573***	1.723***	1.315**	1.085**	1.150**
	(0.150)	(0.120)	(0.136)	(0.550)	(0.425)	(0.548)
Friends in trouble with police in past year	0.509***	0.530***	0.455***	0.548	0.381	0.149
	(0.121)	(0.096)	(0.108)	(0.335)	(0.253)	(0.317)
Victim of crime in past year	0.646***	0.505***	0.489***	0.300	0.249	0.315
	(0.111)	(0.087)	(0.102)	(0.323)	(0.209)	(0.259)
Parents spent time in prison (before 1st interview)	0.738**	0.715**	0.733**	-	-	-
	(0.366)	(0.287)	(0.319)			
Ever expelled (before 1st interview)	0.058	0.509**	0.565**	-	-	-
	(0.343)	(0.231)	(0.258)			
Ever arrested (before 1st interview)	-0.129	0.115	0.250	-	-	-
	(0.188)	(0.137)	(0.152)			
Ever sent to prison (before 1st interview)	0.754	0.842*	0.280	-	-	-
	(0.603)	(0.458)	(0.514)			
Male	0.424***	0.490***	0.534***	-	-	-
	(0.133)	(0.105)	(0.121)			
Age	-0.057*	-0.075***	-0.103***	-0.888	0.034	0.052
	(0.032)	(0.026)	(0.030)	(2.511)	(0.493)	(0.528)
Ever sought help for mental health problems (before 1st interview over age 16)	0.343**	0.379***	0.448***	-	-	-
	(0.137)	(0.106)	(0.118)			
PFA: Derbyshire	-0.094	-0.226	-0.602	-	-	-
	(0.421)	(0.349)	(0.457)			

(continued on following page)

	Theft	Economic Crime	Economic Crime (ex. work and school theft)	Theft	Economic Crime	Economic Crime (ex. work and school theft)
PFA: Devon & Cornwall	-0.958**	-0.618*	-0.618	-	-	-
	(0.452)	(0.331)	(0.409)			
PFA: Essex	-1.300*	-1.676***	-2.032***	-	-	-
	(0.674)	(0.520)	(0.673)			
PFA: North Yorkshire	-0.855	-1.041*	-0.789	-	-	-
	(0.608)	(0.564)	(0.569)			
Sweep 3	0.089	-0.028	-0.167	-	-	-
	(0.187)	(0.148)	(0.172)			
Constant	-2.484***	-1.215*	-1.700**	-	-	-
	(0.934)	(0.732)	(0.843)			
N	**5,650**	**5,650**	**5,650**	**653**	**1,033**	**765**
i	**3,105**	**3,105**	**3,105**	**236**	**377**	**280**
Log likelihood	**-1,461.47**	**-2,066.16**	**-1,607.09**	**-200.02**	**-322.98**	**-231.61**
Median predicted probability of offending report[1]	**0.053**	**0.104**	**0.062**	-	-	-
p-value for joint test of managing on income H_0: =0	**0.608**	**0.987**	**0.925**	**0.670**	**0.698**	**0.941**
p-value for joint test of employment status H_0: =0	**0.397**	**0.878**	**0.999**	**0.946**	**0.566**	**0.842**
p-value for joint test of 'OK to break the law' H_0: =0	**0.000**	**0.000**	**0.000**	-	-	-

Notes: Standard errors are given in parentheses. For the basic logit regression the standard errors are robust to clustering. Significance levels: * 10% significance, ** 5% significance and *** 1% significance. For the fixed effects logit co-efficients are only obtained for time-varying independent variables as all time-invariant variables are conditioned out. Time-invariant variables have co-efficients marked "-". Far lower values of N and i are reported for the fixed effects logit are reported since only individuals with variation in their offending status across time are included in these estimations. The p-values reported test whether the marginal effects are jointly different from zero for the set of independent variables stated. Independent variables which were frequently significant at the 5% level or above in the basic logit estimations but not shown here for brevity are: PFA: South Wales (positive); Wave 2 (positive); Wave 3 (positive); and Sports club/gym member (positive). Many other independent variables were also significant in individual regressions at the 10% level or above.

Table 3.18: Co-efficients from logit and fixed effects logit estimations using the contemporary sample.

One point to note is that the command to implement the fixed effects logit model in Stata does not provide a cluster robust variance-covariance matrix. Hence, the standard errors reported in Table 3.18 for the fixed effects logit estimation are not robust to each individual's error terms being correlated through time. The standard errors reported are likely to be significantly smaller than if this correlation was taken into account. Cameron and Trivedi (2010) suggest this problem can be mitigated by bootstrapping over clusters. Bootstrapping was undertaken with re-sampling occurring 4,000 times; however, convergence of the standard errors did not occur. Yet, for the results in Table 3.18, that the standard errors are biased downwards does not affect the interpretation of the results. If the standard errors increased in size, it would not alter the conclusion that financial position and employment do not show a statistically significant association with offending.

Bivariate Probit Model with Partial Observability

The following description of the bivariate probit model with partial observability is taken from Poirier (1980) with changed notation.

Suppose there are two latent variables: k_i^{**} representing integrity and BC_{it}^* representing the benefit of crime in period t, (BC_{it}^* is akin to the RHS of the NCC). Each of these latent variables can be described as:

$$k_i^{**} = \mathbf{y}_i'\boldsymbol{\gamma}_1 + \varepsilon_{1i}$$

$$BC_{it}^* = \mathbf{x}_{it-1}'\boldsymbol{\beta} + \mathbf{y}_i'\boldsymbol{\gamma}_2 + \varepsilon_{2it}$$

Now suppose that the variable k_i^* represents an individual's integrity type such that:

$$k_i^* = \left\{ \begin{array}{lll} 1 & \text{(low-integrity)} & \text{if } k_i^{**} \leq 0 \\ 0 & \text{(high-integrity)} & \text{if } k_i^{**} > 0 \end{array} \right\}$$

where a high-integrity individual will never offend and a low-integrity individual's offending decision depends on their circumstances. In turn, define BC_{it} as a variable splitting the benefit of crime into high and low categories:

$$BC_{it} = \left\{ \begin{array}{lll} 1 & \text{(high benefit)} & \text{if } BC^*_{it} > 0 \\ 0 & \text{(low benefit)} & \text{if } BC^*_{it} \leq 0 \end{array} \right\}$$

As in a standard bivariate probit model, the error terms for each of the latent variables, ε_{1i} and ε_{2it}, are jointly normally distributed with a correlation coefficient ρ. Where Poirier (1980) and the bivariate probit model with partial observability depart from the standard probit model is that k^*_i and BC_{it} are both unobservable. The only outcome which is observed is O_{it}, i.e. whether or not an individual offends within a given time period. The probability of an individual offending in a given time period is:

$$p_{it} = P\left(O_{it} = 1\right) = P(k^*_i = 1 \cap BC_{it} = 1) = F\left(\mathbf{y}'_i\boldsymbol{\gamma}_1, \mathbf{x}'_{it-1}\boldsymbol{\beta} + \mathbf{y}'_i\boldsymbol{\gamma}_2; \rho\right)$$

whilst the corresponding probability of not offending is:

$$1 - p_{it} = P(k^*_i = 0 \cup BC_{it} = 0) = 1 - F\left(\mathbf{y}'_i\boldsymbol{\gamma}_1, \mathbf{x}'_{it-1}\boldsymbol{\beta} + \mathbf{y}'_i\boldsymbol{\gamma}_2; \rho\right)$$

That not offending occurs when either $k^*_i = 0$ or $BC_{it} = 0$ means an observation of no offending could result from three different situations: $(k^*_i = 0, BC_{it} = 0)$, $(k^*_i = 1, BC_{it} = 0)$ and $(k^*_i = 0, BC_{it} = 1)$. The current chapter's theoretical model suggests that financial position and employment status only affect the offending decision for low-integrity individuals. Hence, there is an issue similar to zero-inflation in count data models, as many people will never offend simply because $k^*_i = 0$. Using the bivariate probit model with partial observability, allows the marginal effects for financial position and employment status to be estimated conditional on being a low-integrity individual, i.e. $k^*_i = 1$.

Problems were encountered running the model described above in Stata. However, imposing the restriction $\boldsymbol{\gamma}_2 = \mathbf{0}$, it was possible to estimate the model for Theft and Economic Crime. Clearly the restriction $\boldsymbol{\gamma}_2 = \mathbf{0}$ is a strong one, as it implies that the time-varying benefits of crime are not influenced by individuals' fixed characteristics. The unconditional average marginal effects and the average marginal effects conditional on $k_i^* = 1$ are reported in Table 3.19.[49]

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[49]Note the time-invariant explanatory variables influence the average marginal effects for BC_{it} even after conditioning on $k_i^* = 1$. This point can be understood by considering the standard definition of conditional probabilities:

$$P\left(BC_{it} = 1 | k_i^* = 1, \mathbf{x}_{it-1}, \mathbf{y}_i\right) = \frac{P\left(BC_{it} = 1 \cap k_i^* = 1 | \mathbf{x}_{it-1}, \mathbf{y}_i\right)}{P\left(k_i^* = 1 | \mathbf{y}_i\right)} = \frac{F\left(\mathbf{y}_i'\boldsymbol{\gamma}_1, \mathbf{x}_{it-1}'\boldsymbol{\beta}; \rho\right)}{F_k\left(\mathbf{y}_i'\boldsymbol{\gamma}_1, \rho\right)}$$

The conditional probability is still a function of $\mathbf{y}_i$. This statement is adapted from Greene's (2008) discussion of the standard bivariate probit model.

Independent Variable	Unconditional Theft	Unconditional Economic Crime	Conditional on k*=1 Theft	Conditional on k*=1 Economic Crime
Strongly agree: sometimes OK to break the law (1st interview)	-0.017	-0.049	0.008	-0.019
	(0.041)	(0.043)	(0.045)	(0.026)
Neither agree/disagree: sometimes OK to break the law (1st interview)	-0.011	-0.027	0.005	-0.010
	(0.018)	(0.019)	(0.025)	(0.012)
Disagree: sometimes OK to break the law (1st interview)	-0.016	-0.028	0.007	-0.010
	(0.017)	(0.018)	(0.038)	(0.012)
Strongly disagree: sometimes OK to break the law (1st interview)	-0.055***	-0.056***	0.029	-0.022
	(0.020)	(0.020)	(0.150)	(0.023)
Ever expelled (before 1st interview)	0.002	0.088**	-0.001	0.031
	(0.101)	(0.044)	(0.044)	(0.032)
Ever arrested (before 1st interview)	0.006	0.035	-0.003	0.013
	(0.021)	(0.024)	(0.019)	(0.017)
Ever sent to prison (before 1st interview)	0.080	0.309***	-0.033	0.096
	(0.205)	(0.083)	(0.242)	(0.099)
Ever committed economic crime (before 1st interview)[1]	0.117***	0.126***	-0.052	0.047
	(0.014)	(0.016)	(0.283)	(0.046)
Male	0.032**	0.051***	-0.016	0.020
	(0.015)	(0.015)	(0.089)	(0.020)
Ever sought help for mental health problems (before 1st interview over age 16)	0.034*	0.068***	-0.016	0.025
	(0.019)	(0.018)	(0.087)	(0.024)
PFA: Derbyshire	-0.098***	-0.125***	0.054	-0.055
	(0.036)	(0.039)	(0.297)	(0.057)
PFA: Devon & Cornwall	-0.067	-0.111***	0.032	-0.047
	(0.045)	(0.040)	(0.179)	(0.047)
PFA: Essex	-0.121***	-0.158***	0.080	-0.080
	(0.041)	(0.037)	(0.446)	(0.078)
PFA: North Yorkshire	-0.111***	-0.153***	0.067	-0.075
	(0.039)	(0.041)	(0.361)	(0.078)
Household just getting by on income	0.008	0.015	0.040	0.039
	(0.016)	(0.014)	(0.064)	(0.038)
Household getting into difficulties on income	0.035	0.048	0.173	0.130
	(0.037)	(0.035)	(0.261)	(0.098)
Respondent employment status: intermediate occupation	-0.016	-0.015	-0.077	-0.038
	(0.022)	(0.027)	(0.141)	(0.069)
Respondent employment status: routine and manual occupations	0.018	0.025	0.091	0.067
	(0.025)	(0.025)	(0.102)	(0.067)
Respondent employment status: looking for paid work/training	-0.057*	-0.040	-0.269	-0.102
	(0.029)	(0.035)	(0.339)	(0.088)

(continued on following page)

	Theft	Economic Crime	Theft	Economic Crime
Taken drugs in past year (not 'Class A')	0.055***	0.097***	0.266	0.261***
	(0.016)	(0.018)	(0.178)	(0.068)
Taken 'Class A' drugs in past year	0.037**	0.102***	0.181	0.273***
	(0.018)	(0.023)	(0.134)	(0.069)
Friends in trouble with police in past year	0.043	0.070***	0.210***	0.188***
	(0.031)	(0.016)	(0.078)	(0.063)
Victim of crime in past year	0.012	0.011	0.059	0.030
	(0.014)	(0.013)	(0.059)	(0.033)
Age	-0.002	-0.004	-0.009	-0.009
	(0.003)	(0.003)	(0.017)	(0.009)
Sweep 3	-0.047*	-0.051**	-0.225	-0.133**
	(0.025)	(0.023)	(0.182)	(0.061)
N	**3,268**	**3,268**	**3,268**	**3,268**
i	**2,004**	**2,004**	**2,004**	**2,004**
Log likelihood	**-838.49**	**-1,126.54**	**-838.49**	**-1,126.54**
Median predicted probability of offending report	**0.048**	**0.095**	**-**	**-**
Median predicted probability of being a criminal type	**-**	**-**	**0.130**	**0.322**
Median predicted conditional probability of reporting offending given respondent is a criminal type	**-**	**-**	**0.446**	**0.323**
p-value for joint test of managing on income H_0: =0	**0.722**	**0.393**	**0.78**	**0.429**
p-value for joint test of employment status H_0: =0	**0.123**	**0.184**	**0.516**	**0.421**
p-value for joint test of 'OK to break the law' H_0: =0	**0.047**	**0.078**	**-**	**-**

Notes: Cluster robust standard errors are given in parentheses. Significance levels: * 10% significance, ** 5% significance and *** 1% significance. The p-values reported test whether the marginal effects are jointly different from zero for the set of independent variables stated. Independent variables which were frequently significant at the 5% level or above in the unconditional regression but not shown here for brevity are: Drinks alcohol 1-3 times a month (positive); PFA: Dyfed Powys (negative); PFA: Wiltshire (negative); and Sports club/gym member (positive). Many other independent variables were also significant in individual regressions at the 10% level or above. The horizontal line in the table indicates the split between independent variables used to estimate a respondent being a potential offender and independent variables used to estimate a respondent committing an offence within a given time period. The variables above the line are used in the potential offender estimation.

[1] This variable varies by dependent variable. If the dependent variable is "Theft" then this variable is whether the respondent has ever committed "Theft" before their first interview.

Table 3.19: Average marginal and conditional average marginal effects for a bivariate probit model with partial observability.

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